$$x = \rho \sin\varphi \cos\theta$$
$$y = \rho \sin\varphi \sin\theta$$
$$z = \rho \cos\varphi$$

$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,\varphi)} = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & \frac{\partial z}{\partial \theta} \\ \frac{\partial x}{\partial \varphi} & \frac{\partial y}{\partial \varphi} & \frac{\partial z}{\partial \varphi} \end{vmatrix} =$$

$$= \begin{vmatrix} \sin\varphi\cos\theta & \sin\varphi\sin\theta & \cos\varphi \\ -\rho\sin\varphi\sin\theta & \rho\sin\varphi\cos\theta & 0 \\ \rho\cos\varphi\cos\theta & \rho\cos\varphi\sin\theta & -\rho\sin\varphi \end{vmatrix} =$$

$$= \cos\varphi\left(\underline{-\rho^2\sin\varphi\cos\varphi}\sin^2\theta \underline{-\rho^2\sin\varphi\cos\varphi}\cos^2\theta\right)$$

$$-0 - \rho\sin\varphi\left(\underline{\rho\sin^2}\varphi\cos^2\theta + \underline{\rho\sin^2\varphi}\sin^2\theta\right) =$$

$$= \cos\varphi(-\rho^2\sin\varphi\cos\varphi) - \rho\sin\varphi(\rho\sin^2\varphi) =$$

$$= \rho^2\sin\varphi(-\cos^2\varphi - \sin^2\varphi) =$$

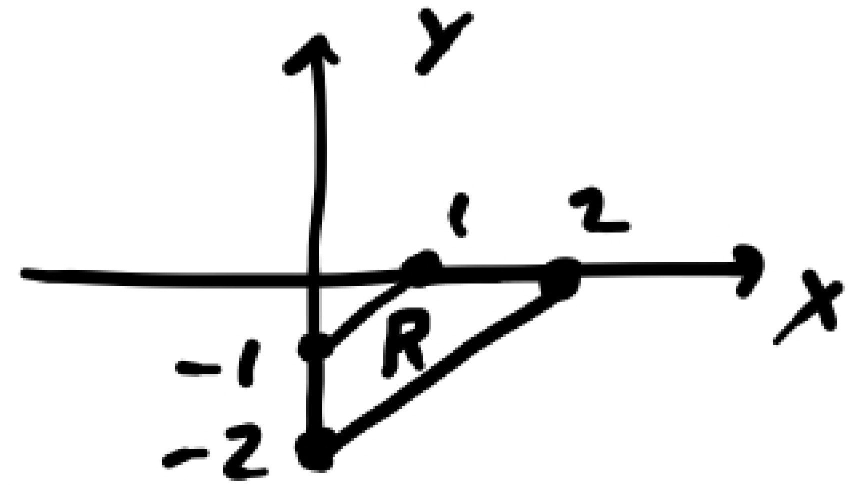
$$= -\rho^2\sin\varphi$$

- If Jacobian is negative, take <u>absolute value</u>.

$$|-\rho^2\sin\varphi| = \rho^2\sin\varphi$$

__Example__ Find $\iint_R e^{\frac{x+y}{x-y}}\, dA$

where R

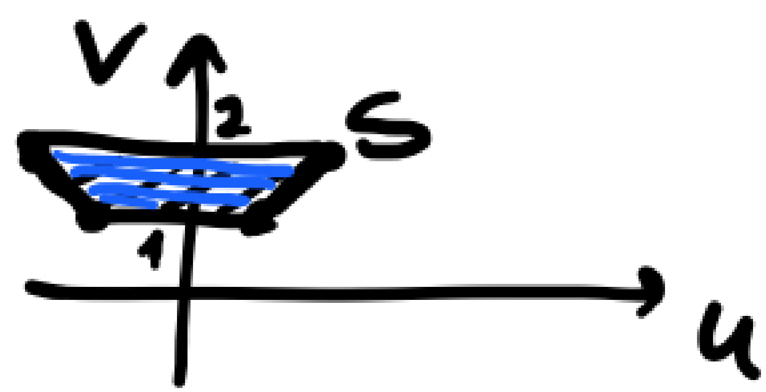


$T^{-1}\begin{cases} u = x+y \\ v = x-y \end{cases}$

Motivation:

- $e^{(x+y)/(x-y)} = e^{u/v}$
- $\int e^{u/v}\,du = v\,e^{u/v} + C$

$T\begin{cases} x = \dfrac{u+v}{2} \\ y = \dfrac{u-v}{2} \end{cases}$



$\dfrac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{vmatrix} = -1/2 \xrightarrow{|\cdot|} 1/2$

$$\iint_R e^{\frac{x+y}{x-y}}\,dA = \iint_S e^{u/v}\,|J|\,dA =$$

$$= \int_1^2 \int_{-v}^{v} e^{u/v}\ \frac{1}{2}\,du\,dv =$$

$$= \int_1^2 \frac{1}{2} v e^{u/v}\Big|_{-v}^{v}\,dv =$$

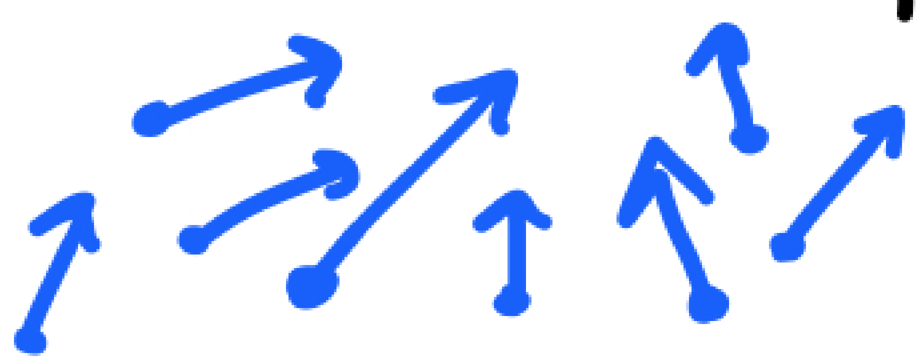
$$= \int_1^2 \frac{1}{2} v\,(e^1 - e^{-1})\,dv$$

$$= \frac{1}{2}(e-e^{-1})\,\frac{v^2}{2}\Big|_1^2 = \frac{3}{4}(e-e^{-1})$$

Vector Calculus

16.1. Vector Fields (2D)

- Vector field is a function that assigns vectors to points in space.



e.g. velocity vector field

records direction and speed of particles

Def. let D be a domain in $\mathbb{R}^2$.
A vector field is a function

F that assigns to each pt (x,y)
in D a two-dim. vector $F(x,y)$.

$$F(x,y) = P(x,y)\,\hat{\imath} + Q(x,y)\,\hat{\jmath}$$

usual function