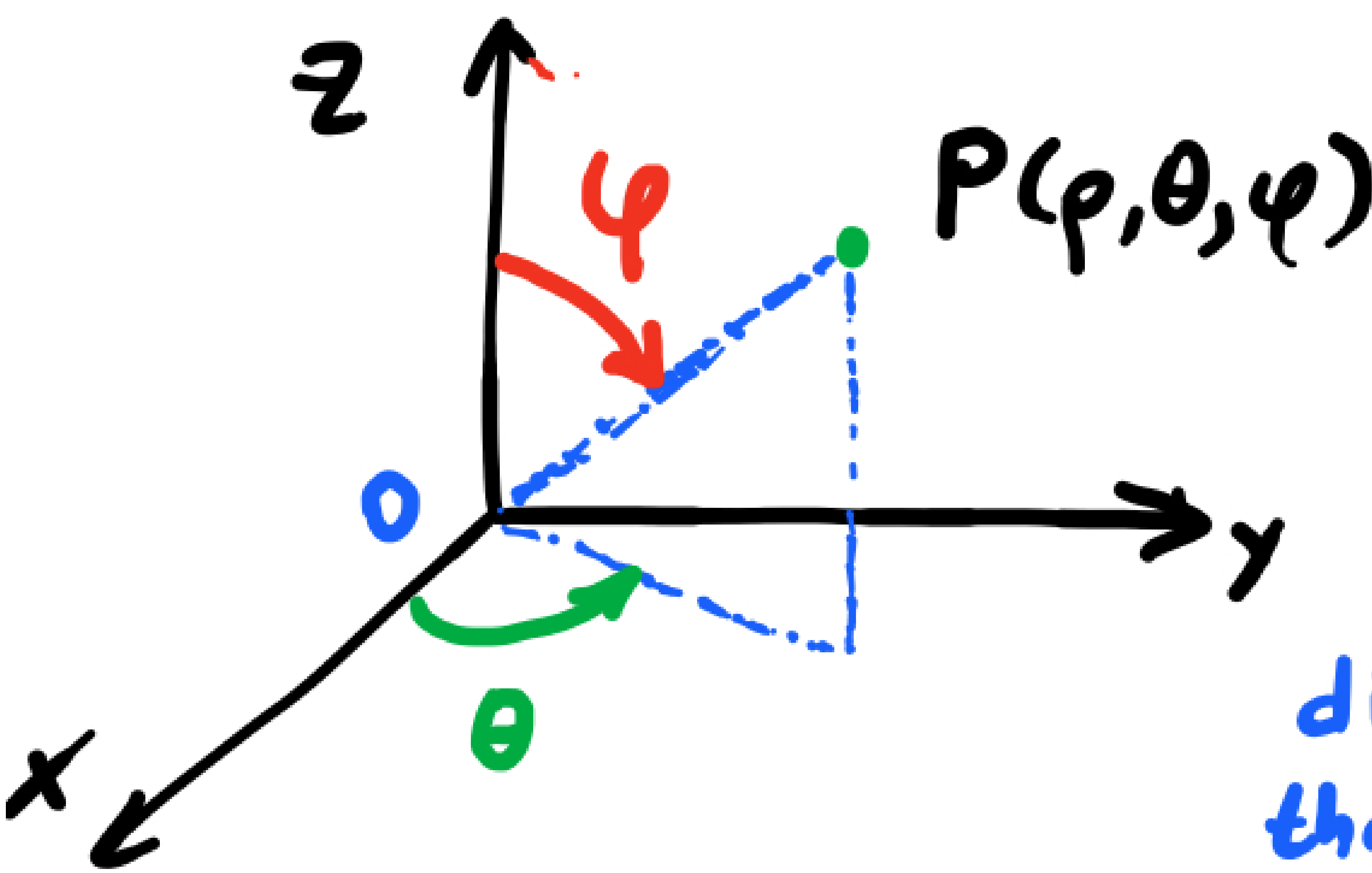
15.8. Triple Integrals in Spherical coordinates.



$$(\rho, \theta, \varphi)$$

ρ: distance from the origin

θ: same as θ in cylindr. coordinates

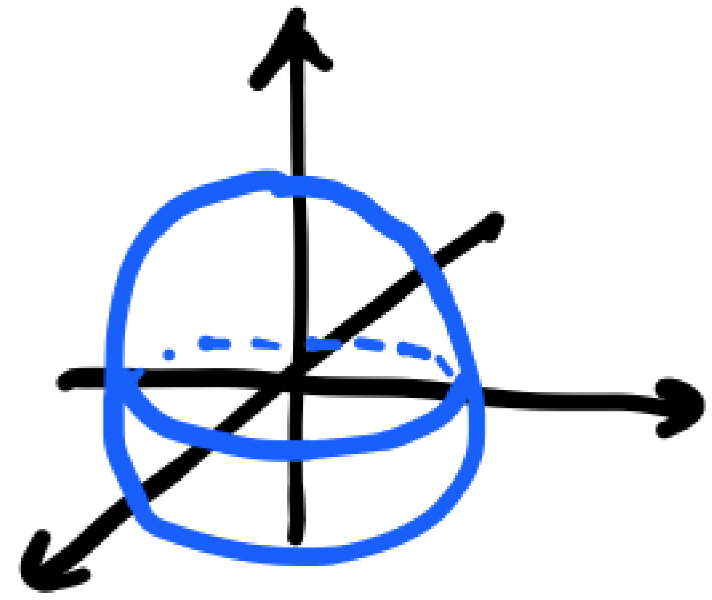
φ: angle between positive z-axis and OP

We have $\rho \geq 0$, $0 \leq \theta \leq 2\pi$, $0 \leq \varphi \leq \pi$

Surfaces

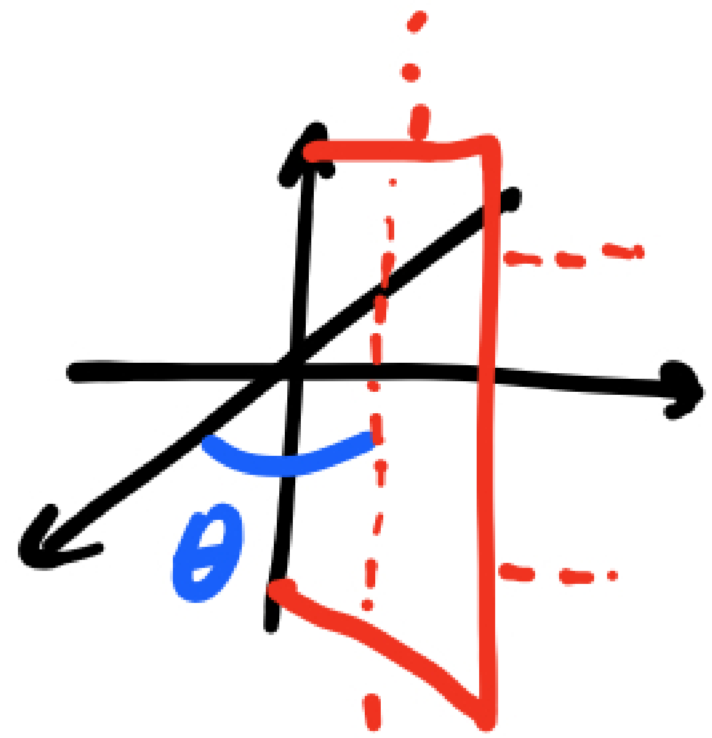
$\rho = \text{const}$

a sphere



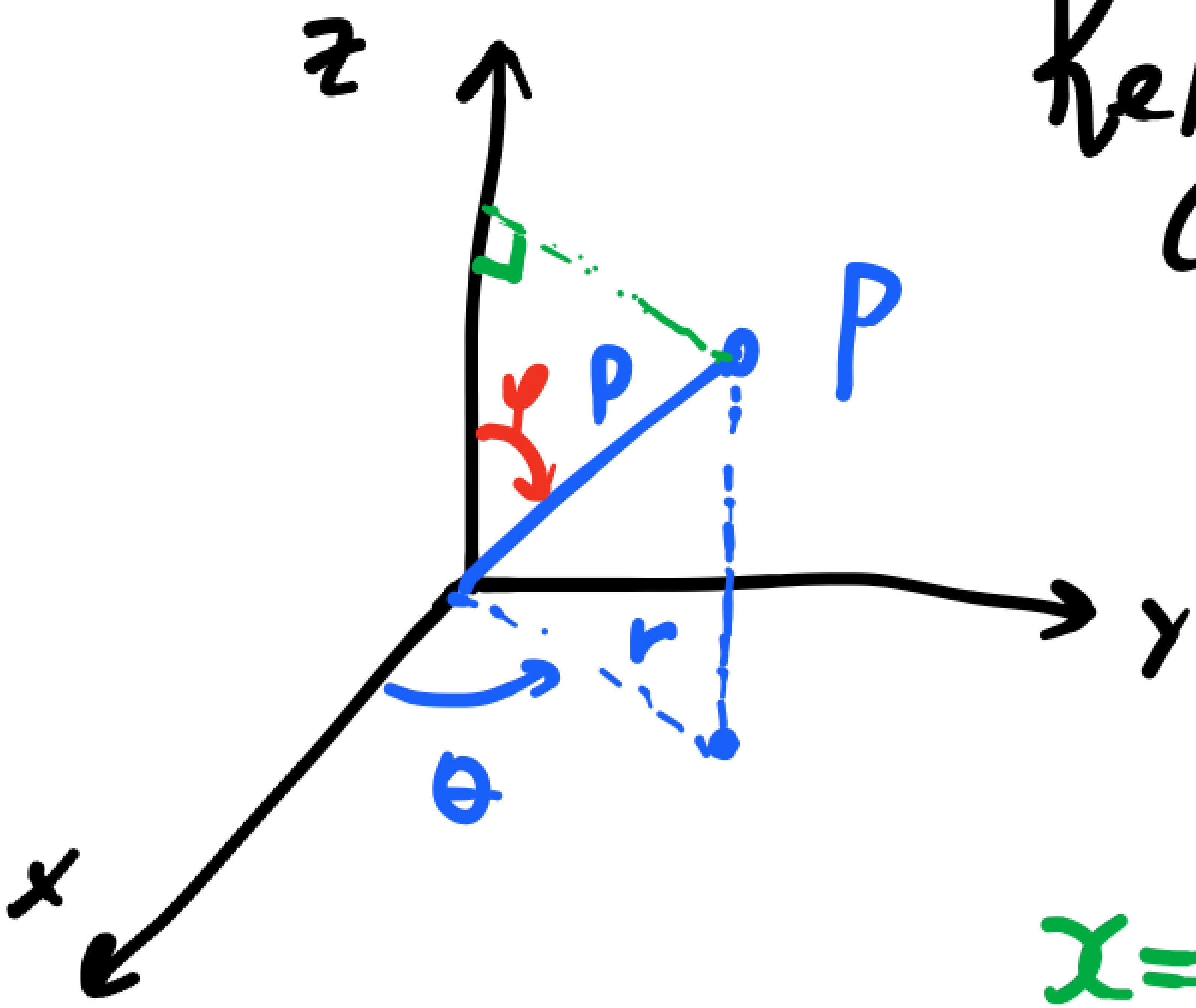
$\theta = \text{const}$

a half-plane



$\varphi = \text{const}$

a half-cone



Relationship with Cartesian coordinates:

$$z = \rho \cos\varphi$$

$$r = \rho \sin\varphi$$

$$x = r\cos\theta \qquad y = r\sin\theta$$

$$\boxed{x = \rho \sin\varphi \cos\theta \qquad y = \rho \sin\varphi \sin\theta \qquad z = \rho\cos\varphi}$$

$$\boxed{\rho^2 = x^2 + y^2 + z^2}$$

Examples

- convert $\left(3, \frac{\pi}{6}, \frac{\pi}{3}\right)$ into cartesian coordinates
- convert $(2, 0, -1)$ into spherical coord.

$x = 3 \cdot \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{9}{4}$ $\quad y = 3 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = \frac{3\sqrt{3}}{4}$ $\quad z = 3 \cdot \frac{1}{2} = \frac{3}{2}$

$\rho = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{5}$

$\cos\varphi = \frac{z}{\rho} = -\frac{1}{\sqrt{5}}$

$\varphi = \arccos\left(-\frac{1}{\sqrt{5}}\right)$

$\cos\theta = \frac{x}{\rho \sin\varphi} = \frac{2}{\sqrt{5} \cdot \frac{2}{\sqrt{5}}} = 1$

$\cos^2\varphi + \sin^2\varphi = 1$

$1/5 + \sin^2\varphi = 1$

$\sin^2\varphi = 4/5$

$\sin\varphi = \pm 2/\sqrt{5}$

$\theta = 0.$

General remark there is no universal agreement on the notation (ρ, θ, φ) for spherical coordinates

e.g. In physics (r, φ, θ) is often used

Next time: triple integrals in sph. coordinates.