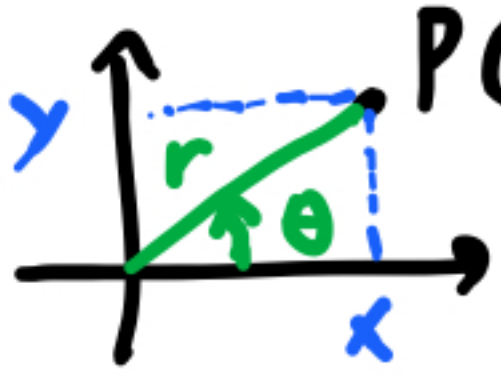
Triple in cylindrical coordinates.

2D: Cartesian coordinates (x,y), polar coordinates (r,θ)

3D: Cartesian coordinates (x,y,z),

cylindrical coordinates (r,θ,z),
spherical coordinates
(ρ,θ,φ)

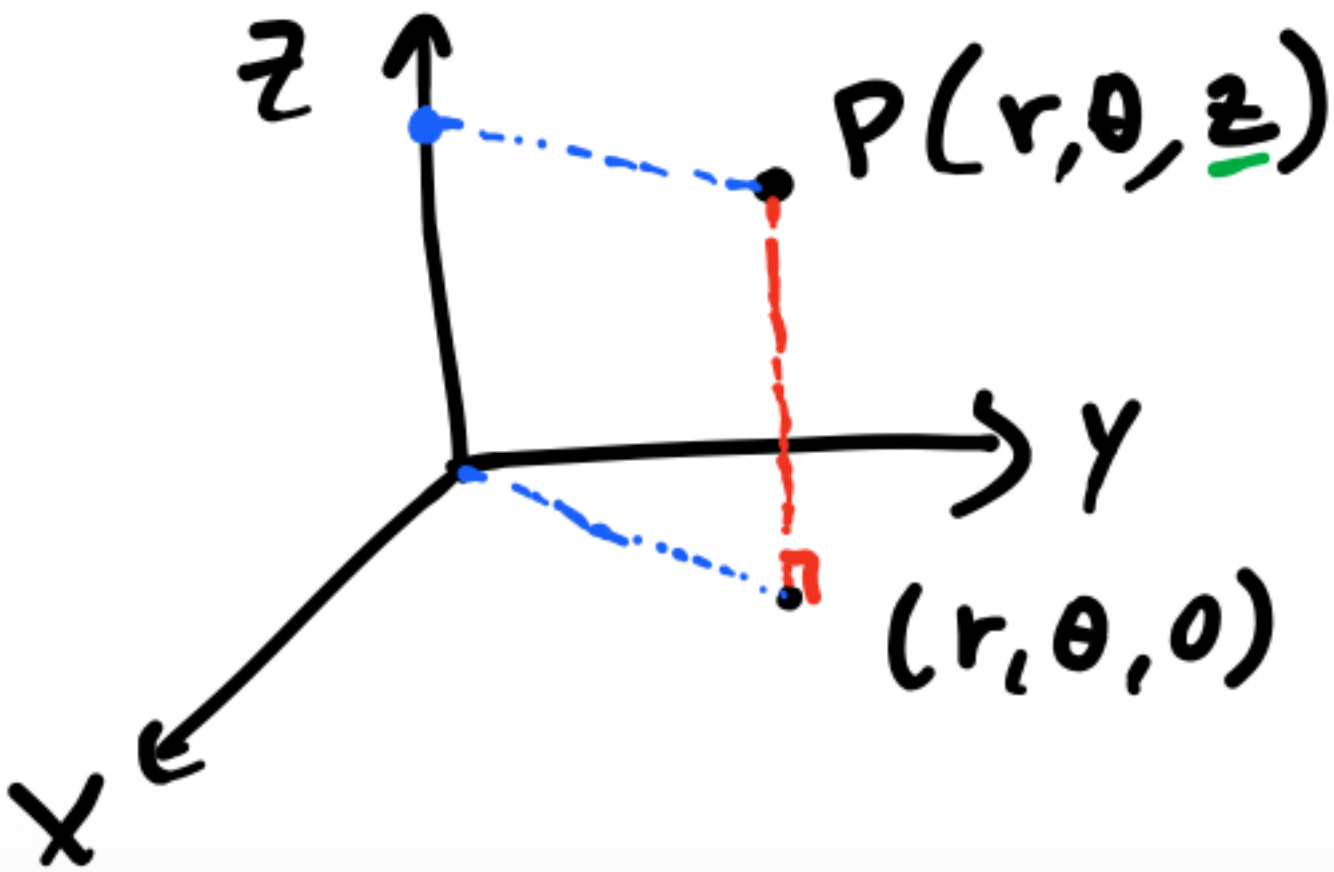
polar coordinates



$P(r,\theta) = P(x,y)$

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2 + y^2$$
$$\tan\theta = y/x$$

cylindrical coordinates



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$
$$r^2 = x^2 + y^2$$
$$\tan\theta = y/x$$
$$z = z$$

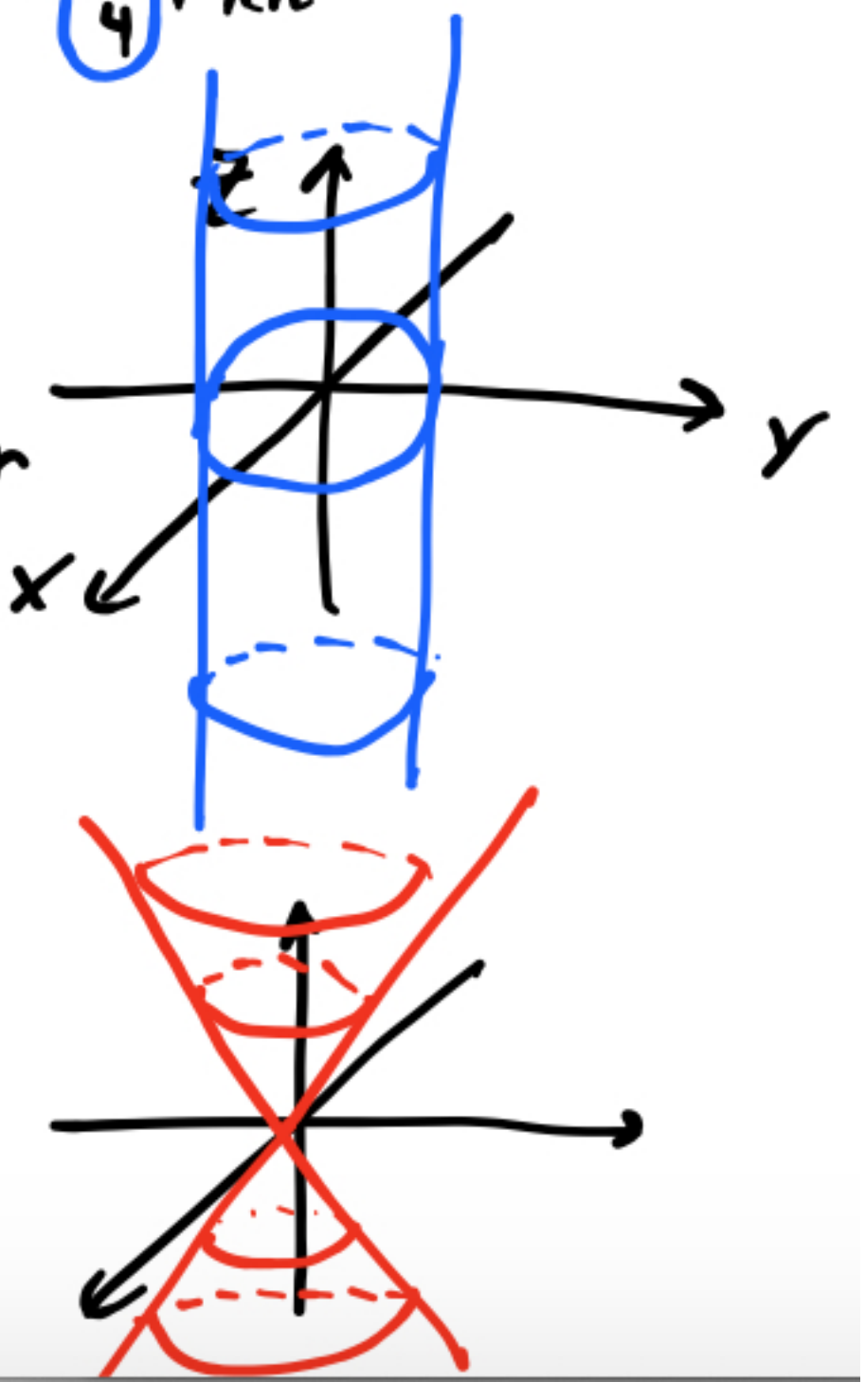
Example · convert $(2, \pi/3, -1)$ into cartesian coord.

· convert $(2, 2, 5)$ into cylindr. coord.

$\rightarrow$ $x = 2\cos \pi/3 = 1$, $y = 2\sin\frac{\pi}{3} = \sqrt{3}$, $z = -1$.

$\rightarrow$ $r = \sqrt{2^2+2^2} = 2\sqrt{2}$, $\tan\theta = 1$, $z = 5$

$\theta = \frac{\pi}{4} + \pi n$



Surfaces · $r = \text{const}$

infinite cylinder

· $z = r$

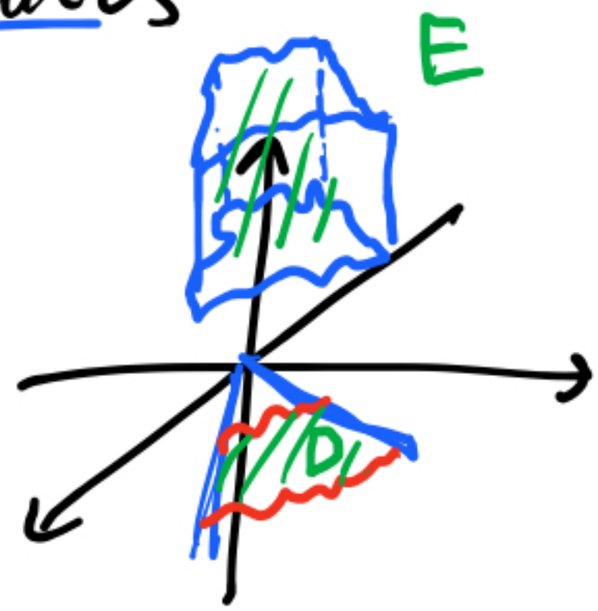
double cone

$z^2 = r^2 = x^2 + y^2$

Triple integrals in cylindrical coordinates

Suppose E is type 1

$$E = \{(x,y,z) \mid (x,y) \in D,\ u_1(x,y) \le z \le u_2(x,y)\}$$



and D is given by

$$D = \{(r,\theta) \mid \alpha \le \theta \le \beta,\ h_1(\theta) \le r \le h_2(\theta)\}$$

Then

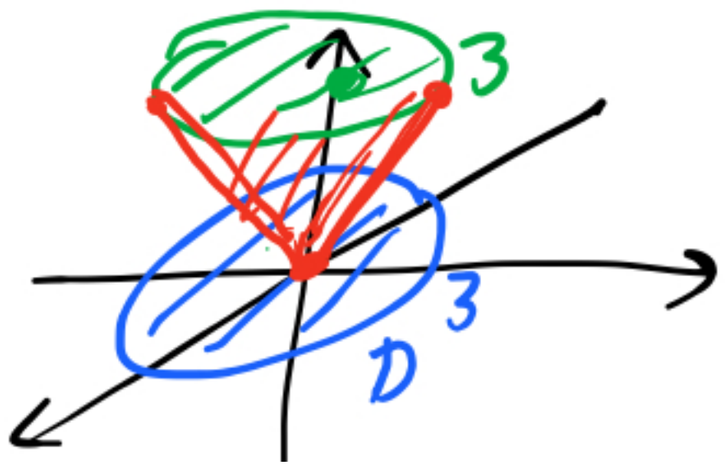
$$\iiint_E f(x,y,z)\,dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r\cos\theta, r\sin\theta)}^{u_2(r\cos\theta, r\sin\theta)} f(r\cos\theta, r\sin\theta, z)\; r\,dz\,dr\,d\theta$$

Examples Find $I = \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{\sqrt{x^2+y^2}}^{3} (x^2+y^2)\, dz\, dy\, dx$

- Express E

$$E = \{ (x,y,z) \mid \underbrace{-3 \le x \le 3,\ -\sqrt{9-x^2} \le y \le \sqrt{9-x^2}}_{D},\ \underbrace{\sqrt{x^2+y^2} \le z \le 3}_{r} \}$$

- draw E



$x^2+y^2=9$

- re-express E in cylindrical coordinates

$$E = \{(r,\theta,z) \mid \begin{array}{l} 0 \le r \le 3 \\ 0 \le \theta \le 2\pi \\ r \le z \le 3 \end{array} \}$$

- Compute integral

$$I = \iiint_E x^2+y^2 \, dV = \underbrace{\int_0^{2\pi}\int_0^3\int_r^3 r^2 \cdot r \, dz \, dr \, d\theta}_{\text{exercise}}$$