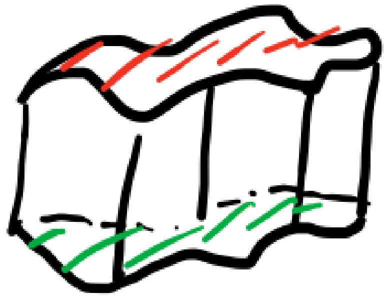
Applications of double integrals

- volumes of solids



- surface area (later)

e.g. $f = \sqrt{1-x^2-y^2}$

- physics
  - computing mass
  - center of mass
  - electric charge

  ⋮

probability
- density function of 2 random variables

Density and Mass

Lamina (thin sheet) occupies a region D and its density (in mass per unit area) at (x,y) is given by $\rho(x,y)$

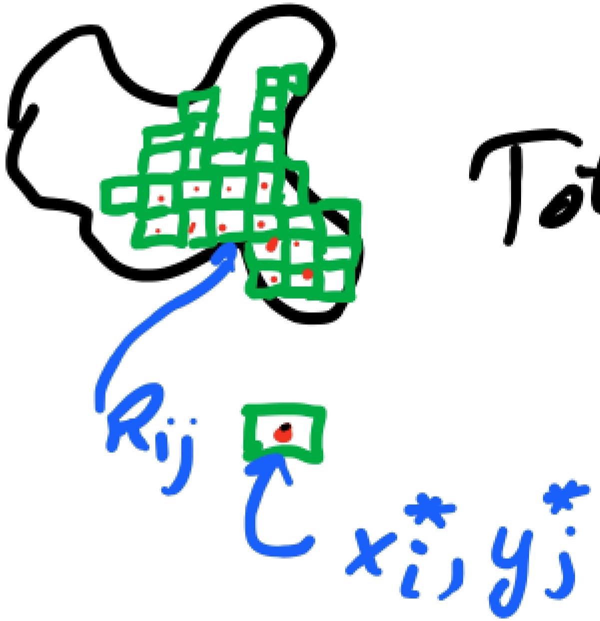
$$\rightarrow \quad \rho(x,y) = \lim_{\square \to 0} \frac{\Delta m}{\Delta A}$$

size of rectangle

Δm – mass of small rectangle

ΔA - its area

size of rectangle ΔA - its area

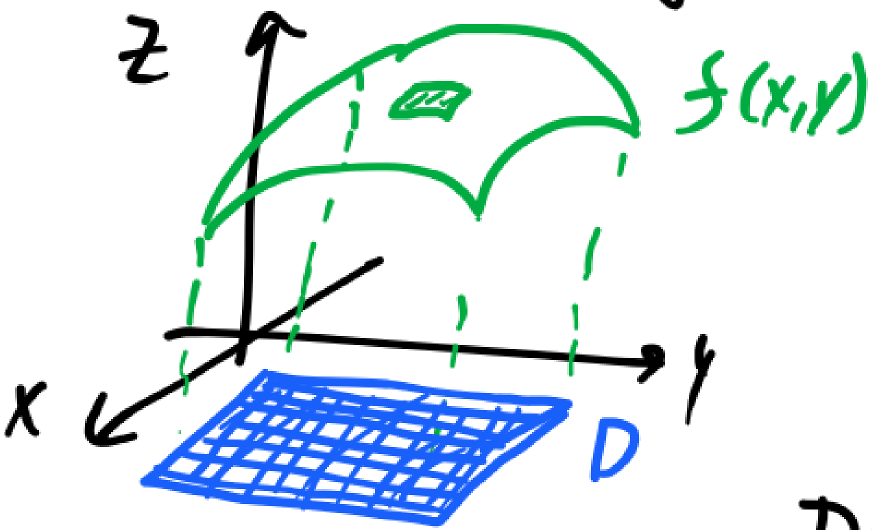


Total mass of lamina:

$$m \approx \sum_{i=1}^{k} \sum_{j=1}^{l} \underbrace{\rho(x_i^*, y_j^*)\Delta A}_{\text{approx. mass of } R_{ij}}$$

$$m = \lim_{k,l \to \infty} (\ldots) = \iint_D \rho(x,y)\, dA$$

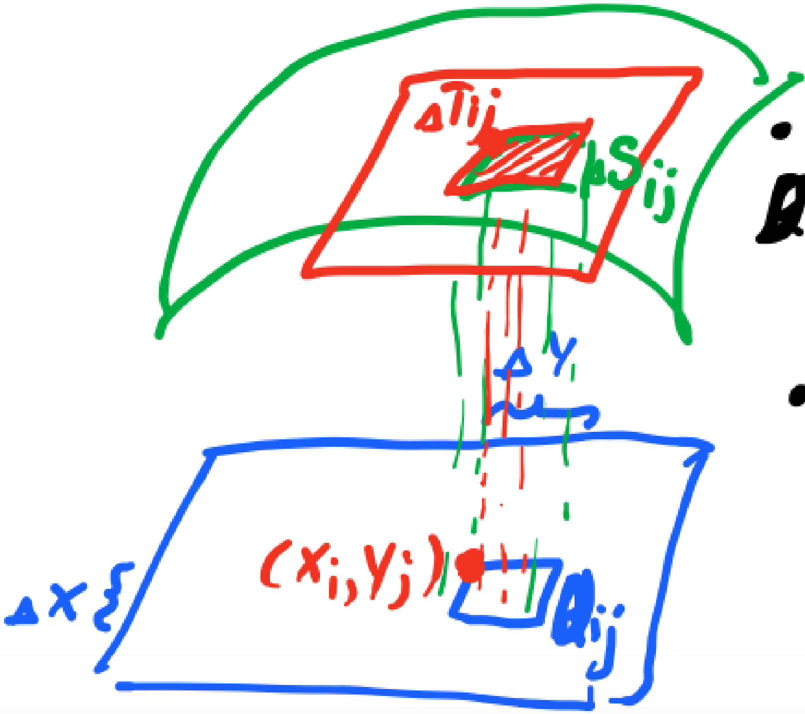
15.5. Surface area



let S be a surface with equation

$$z = f(x,y)$$

Domain of $f(x,y)$: rectangle D.



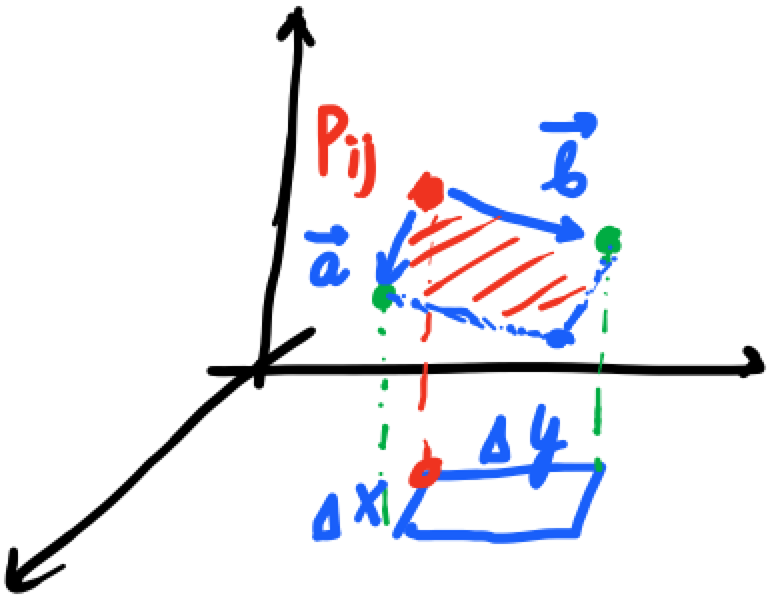
- divide D into small rectangles D_{ij} of area $\Delta A = \Delta x \cdot \Delta y$
- approximate the area ΔS_{ij} of the part of S above D_{ij} by the area ΔT_{ij} of the part of the tangent plane

(a parallelogram) at point

$P_{ij} = (x_i, y_j, f(x_i, y_j))$ above

D_{ij}.

Formula for ΔT_{ij}



Recall: $\Delta T_{ij} =$

$$= \| a \times b \|$$

cross product

"slope" of a : f_x

"slope" of b : f_y

$$a = \underbrace{\Delta x}\, \hat{i} + \underbrace{f_x(x_i, y_j)\,\Delta x}\, \hat{k}$$

$$b = \Delta y\, \hat{j} + f_y(x_i, y_j)\,\Delta y\, \hat{k}$$

$$a \times b = \begin{vmatrix} i & j & k \\ \Delta x & 0 & f_x \Delta x \\ 0 & \Delta y & f_y \Delta y \end{vmatrix} =$$

$$= -f_x \Delta A \,\hat{\imath} - f_y \Delta A \,\hat{\jmath} + \Delta A \,\hat{k}$$

$$= \left[-f_x \hat{\imath} - f_y \hat{\jmath} + k \right] \Delta A$$

$$\Delta T_{ij} = \\ = \| a \times b \| = \underbrace{\sqrt{(f_x)^2 + (f_y)^2 + 1}} \; \Delta A$$

$$A(S) = \lim_{m,n \to \infty} \sum \sum \Delta T_{ij} =$$

$$= \iint_D \sqrt{f_x^2 + f_y^2 + 1} \, dA$$