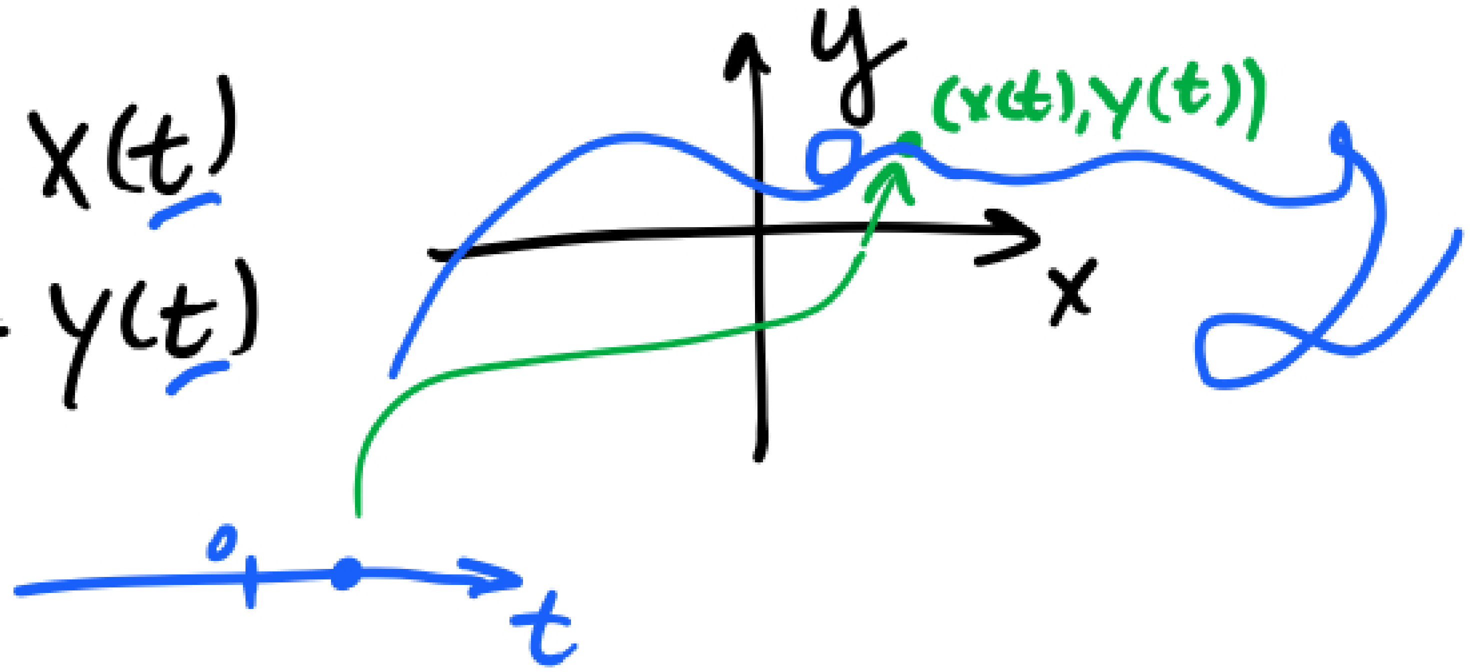
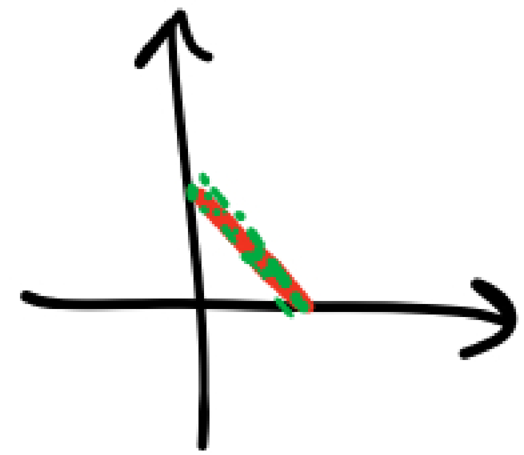
$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$



$$\begin{cases} x = \cos^2 t \\ y = \sin^2 t \end{cases}$$

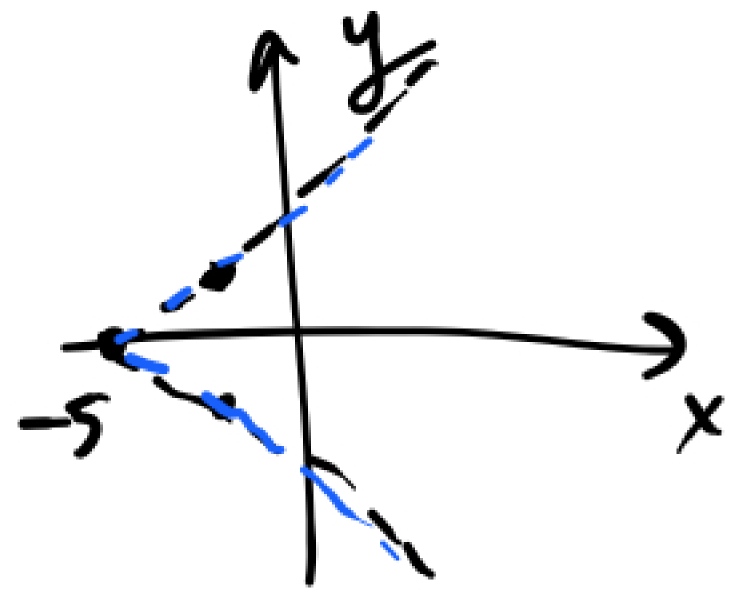
$$x + y = 1.$$

$$y = 1 - x$$



Example

$$\begin{cases} x = 2t^2 - 5 \\ y = t^3 + t \end{cases}$$



• plug points

$t = 0 \dashrightarrow (-5, 0)$

$t = 1 \dashrightarrow (-3, 2)$

$t = -1 \dashrightarrow (-3, -2)$

- Solve for t

$$t^2 = \frac{x+5}{2} \Rightarrow t = \pm\sqrt{\frac{x+5}{2}}$$

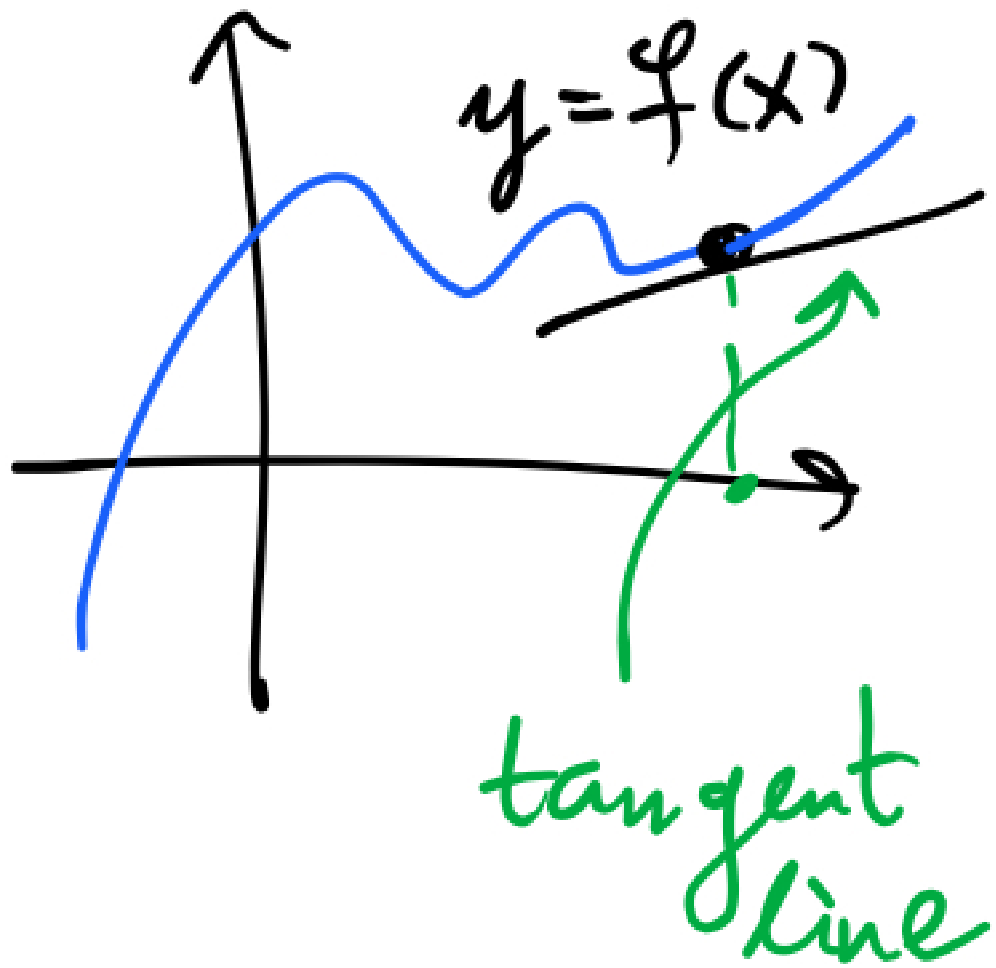
$$\Rightarrow y = \left(\pm\sqrt{\frac{x+5}{2}}\right)^3 + \left(\pm\sqrt{\frac{x+5}{2}}\right).$$

Upshot: plotting parametric curves may be not easy.

Use software!

10.2. Calulus with Parametric Curves.

$$\begin{cases} x = x(t) \\ y = y(t) \end{cases}$$



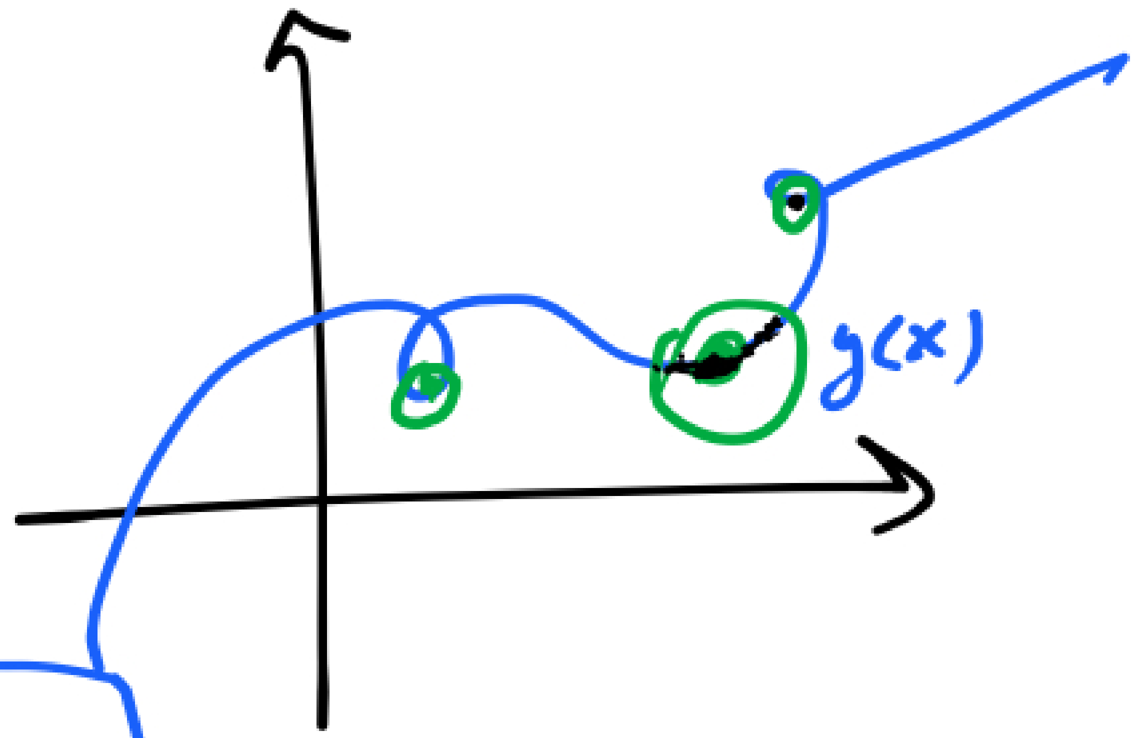
Slope of the tangent line $= \dfrac{dy}{dx}$

$$\frac{dy(x(t))}{dt} \underset{\text{rule}}{\overset{\text{chain}}{=}} \frac{dy}{dx} \cdot \frac{dx}{dt}$$

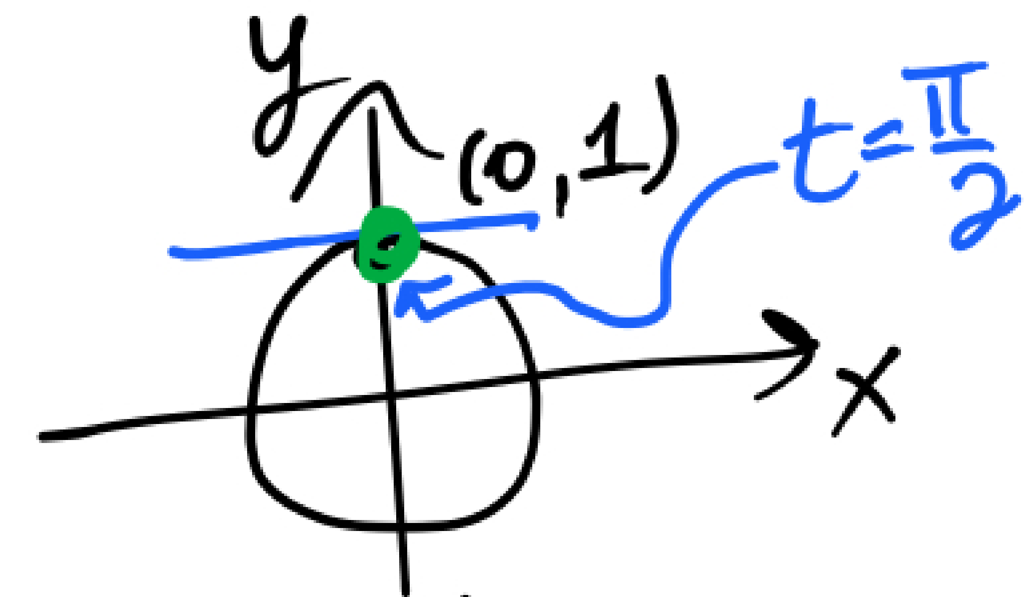
$$/\frac{dx}{dt}$$

$$\Rightarrow$$

$$\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt}}\ \boxed{\text{if } \frac{dx}{dt} \neq 0}$$



Example unit circle



$$\begin{cases} x = \cos t \\ y = \sin t \end{cases} \qquad 0 \le t \le 2\pi$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{-\sin t}$$

At $(0,1)$, $t = \pi/2$ $\qquad \frac{dy}{dx} = \frac{0}{-1} = 0$

$\Rightarrow$ tangent line is horizontal.

Upshot: $\frac{dy}{dt} = 0, \frac{dx}{dt} \neq 0 \Rightarrow$ horizontal tangent

$\frac{dx}{dt} = 0, \frac{dy}{dt} \neq 0 \Rightarrow$ vertical tangent

$\frac{dx}{dt} = 0, \quad \frac{dy}{dt} = 0 \Rightarrow$ no colclusion (for now)

Example (from before) $\begin{cases} x = 2t^2 - 5 \\ y = t^3 + t \end{cases}$ $t \in \mathbb{R}$ all real numbers

(1) Are there pts where the tangent line is horizontal? vertical?

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2+1}{4t}$$

$3t^2+1 \to$ never zero

$4t$: $t=0$

- no horizontal tangent line
- vertical tangent at $t=0$.

(2) Are there pts where pts where tangent line is perp to $y=-x-3$?
perpendicular

Lines with slopes a_1, a_2 are perp

$\Leftrightarrow a_1 \cdot a_2 = -1$

so $\frac{dy}{dx} \cdot (-1) = -1$

$\Leftrightarrow \frac{3t^2+1}{4t} = 1 \Leftrightarrow 3t^2+1 = 4t$

$\Leftrightarrow (3t-1)(t-1) = 0$

$\Leftrightarrow t = 1/3,\ t = 1.$

Yes, $(-3,2)$, $(\ ---\ )$

__Example__ Where is the curve

$$\begin{cases} x = \ln t \\ y = e^t \end{cases} \quad \text{concave up?}$$

$$t > 0$$

Need $\dfrac{d^2y}{dx^2} > 0$.

So we need $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) =$

$$= \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} \quad \left(\text{if } \frac{dx}{dt} \neq 0\right)$$

$$\frac{d^2y}{dx^2} =$$ check!