Warm-up:

$$\iint_R x\cos(2xy)\,dA \text{, where } R = [\quad]\times[\quad]$$

Hint: Fubini? appropriate order of

$[0,\pi]\times[1,3]$

$$\int_1^3\left[\int_0^\pi x\cos(2xy)\,dx\right]dy$$

$$\int_0^\pi\left[\int_1^3 x\cos(2xy)\,dy\right]dx = \int_0^\pi\left[\frac{1}{2}\sin(2xy)\Big|_1^3\right]dx$$

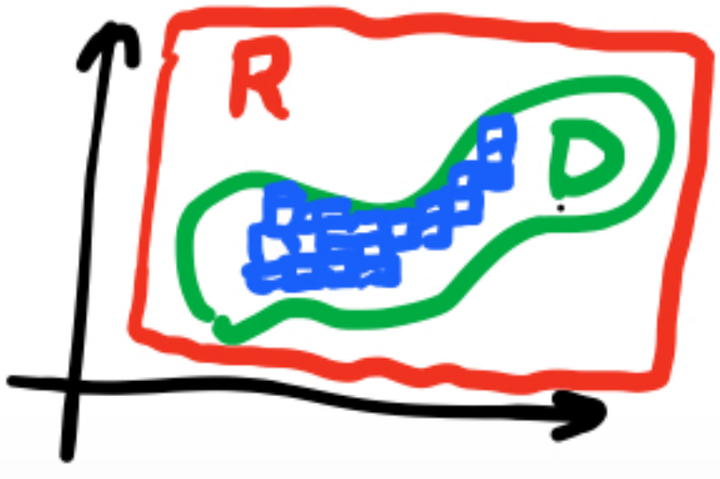
easier

$$= \int_0^{\pi} \left[\frac{1}{2}\sin(6x) - \frac{1}{2}\sin(2x) \right] dx =$$

$$= \left. -\frac{1}{12}\cos(6x) + \frac{1}{4}\cos(2x) \right|_0^{\pi} =$$

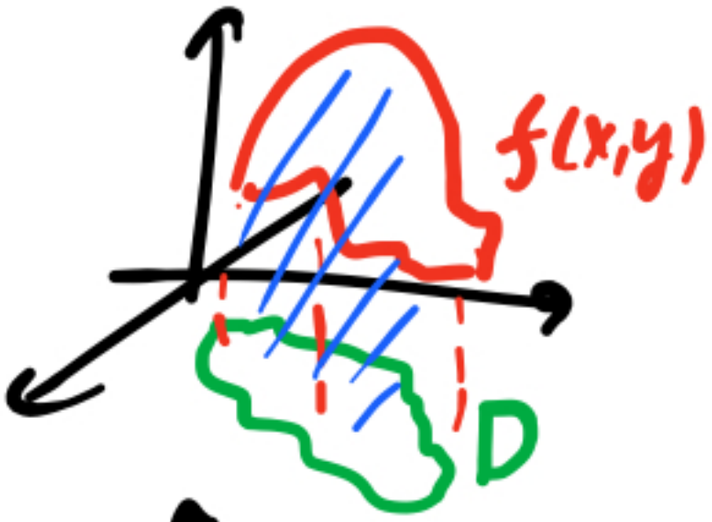
$$= 0.$$

15.2. Double Integrals over General Regions.

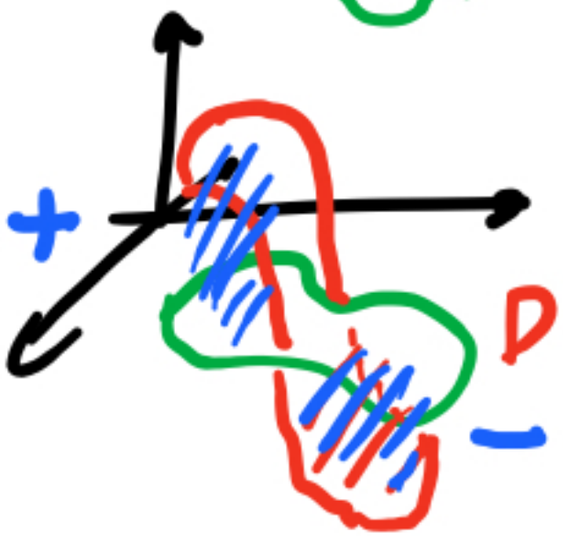


D - domain of $f(x,y)$

$$\iint f(x,y)\,dA = ?$$



$\iint_D$ is same thing (signed) volume under the graph

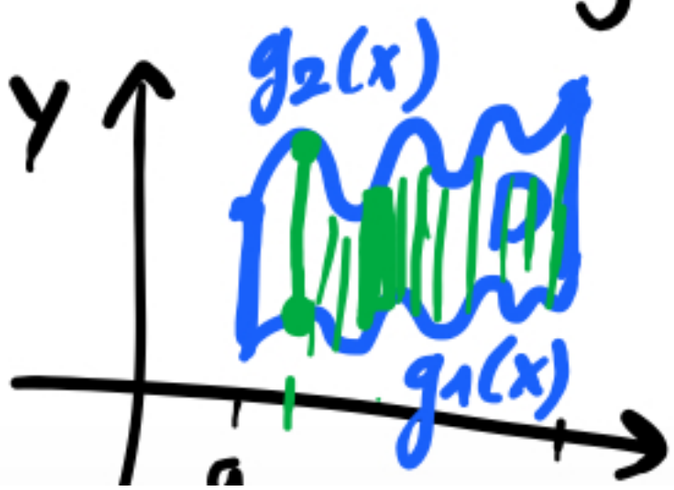


$$\iint_D f(x,y)\,dA \overset{\text{define}}{=} \iint_R F(x,y)\,dA$$

$$F = \begin{cases} f(x,y), & (x,y)\in D \\ 0, & (x,y)\notin D \end{cases}$$

Nice types of regions:

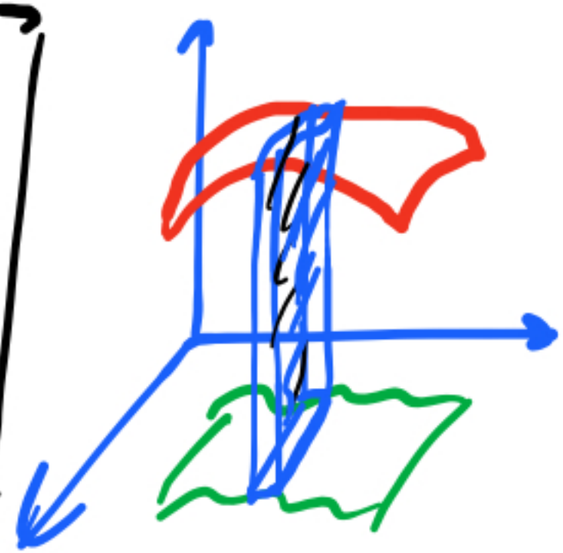
Type I.



$$D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

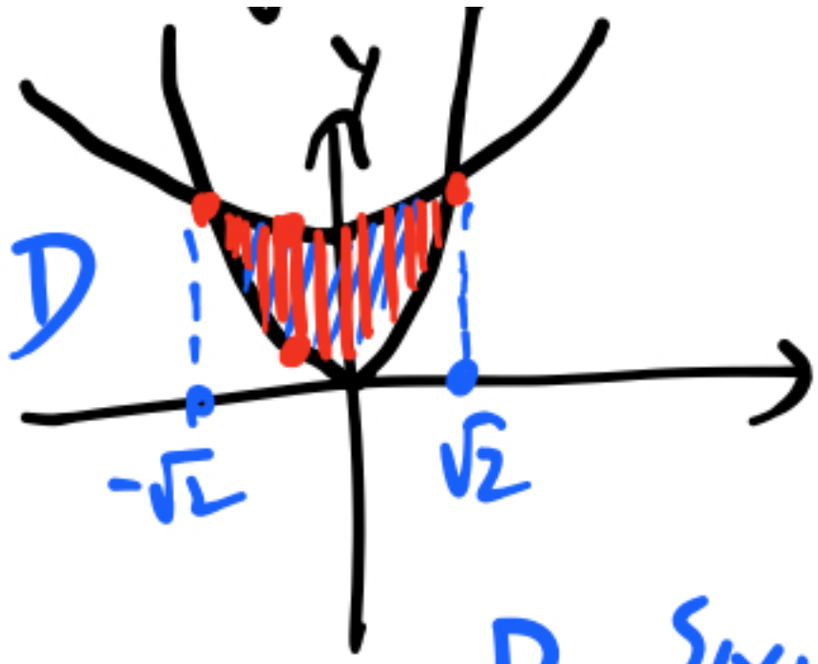
Fubini

$$\iint_D f(x,y)\,dA = \int_a^b \left[\int_{g_1(x)}^{g_2(x)} f(x,y)\,dy \right] dx$$



Example $\iint_D (x-y)\,dA$, where

D is the region bounded by $y = 3x^2$ and $y = 2+2x^2$.



$$3x^2 = 2 + 2x^2$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

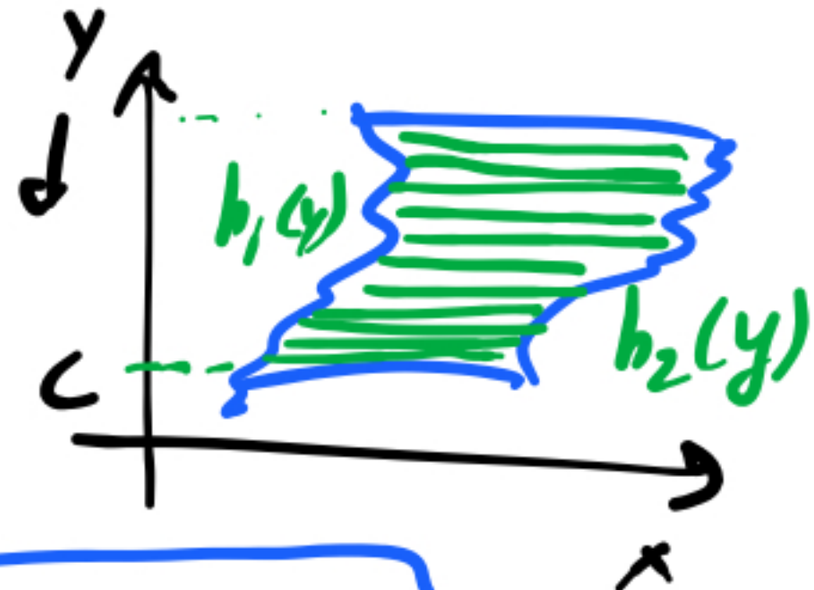
$$D = \{(x,y) \mid -\sqrt{2} \le x \le \sqrt{2},\ 3x^2 \le y \le 2 + 2x^2\}.$$

$$\iint_D (x-y)\,dA = \int_{-\sqrt{2}}^{\sqrt{2}} \left[\int_{3x^2}^{2+2x^2} (x-y)\,dy \right] dx =$$

$$= \int_{-\sqrt{2}}^{\sqrt{2}} \left[xy - \frac{y^2}{2} \Big|_{3x^2}^{2+2x^2} \right] dx =$$

$$= \text{...Exercise...}$$

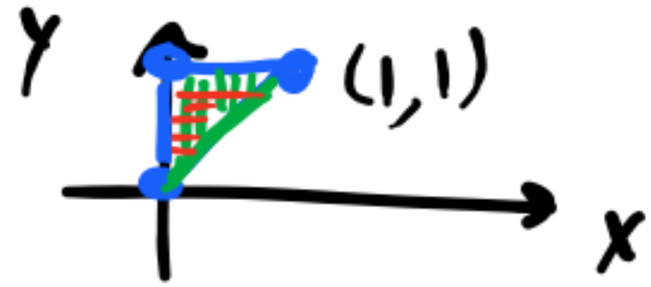
Type II



$$D = \{(x,y) \mid c \le y \le d,\ h_1(y) \le x \le h_2(y)\}$$

$$\iint_D f(x,y)\,dA = \int_c^d \left[\int_{h_1(y)}^{h_2(y)} f(x,y)\,dx \right] dy$$

Remark: D can be both Type I and Type II!



$$D = \{(x,y) \mid 0 \le x \le 1,\ x \le y \le 1\}$$

$$D = \{(x,y) \mid 0 \le x \le y,\ 0 \le y \le 1\}$$