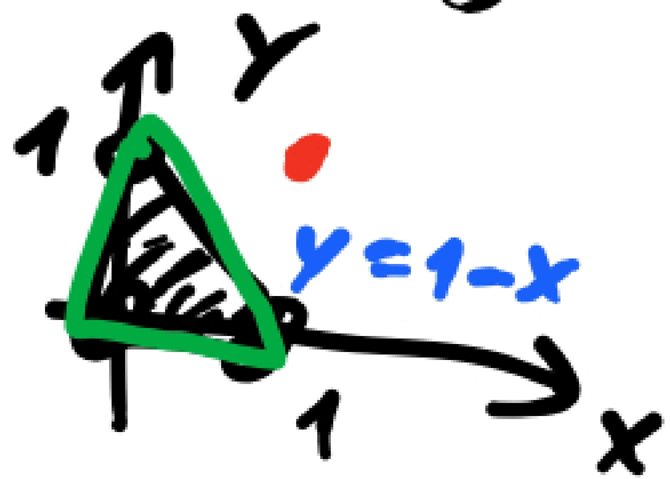
Exercise. max/min $f(x,y) = 3x + 3y - x^2 - y^2 - xy$

on



(1) Crit pts: $\nabla f = \vec{0}$

$$\nabla f = (3 - 2x - y, 3 - 2y - x) = \vec{0}$$

$$\begin{cases} 3 - 2x - y = 0 \\ 3 - 2y - x = 0 \end{cases} \Leftrightarrow (x,y) = (1,1)$$

not in the domain

(2) Along L_1

domain

② Along bottom ($y=0$): $f = 3x - x^2$,

$0 \leq x \leq 1$

Along vertical ($x=0$): $f = 3y - y^2$,

$0 \leq y \leq 1$

Along diagonal ($y = 1-x$): $f = \ldots =$

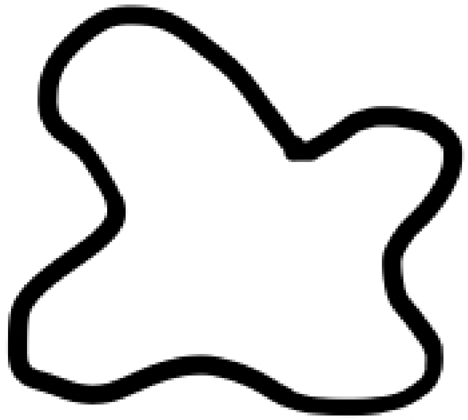
$= -x^2 + x + 2$

$0 \leq x \leq 1$.

Check.

What to do if the boundary is complicated?

e.g. $x^4 + xy + y^4 = 1$



$\cancel{x = x(t)}$

$\cancel{y = y(t)}$

Problem

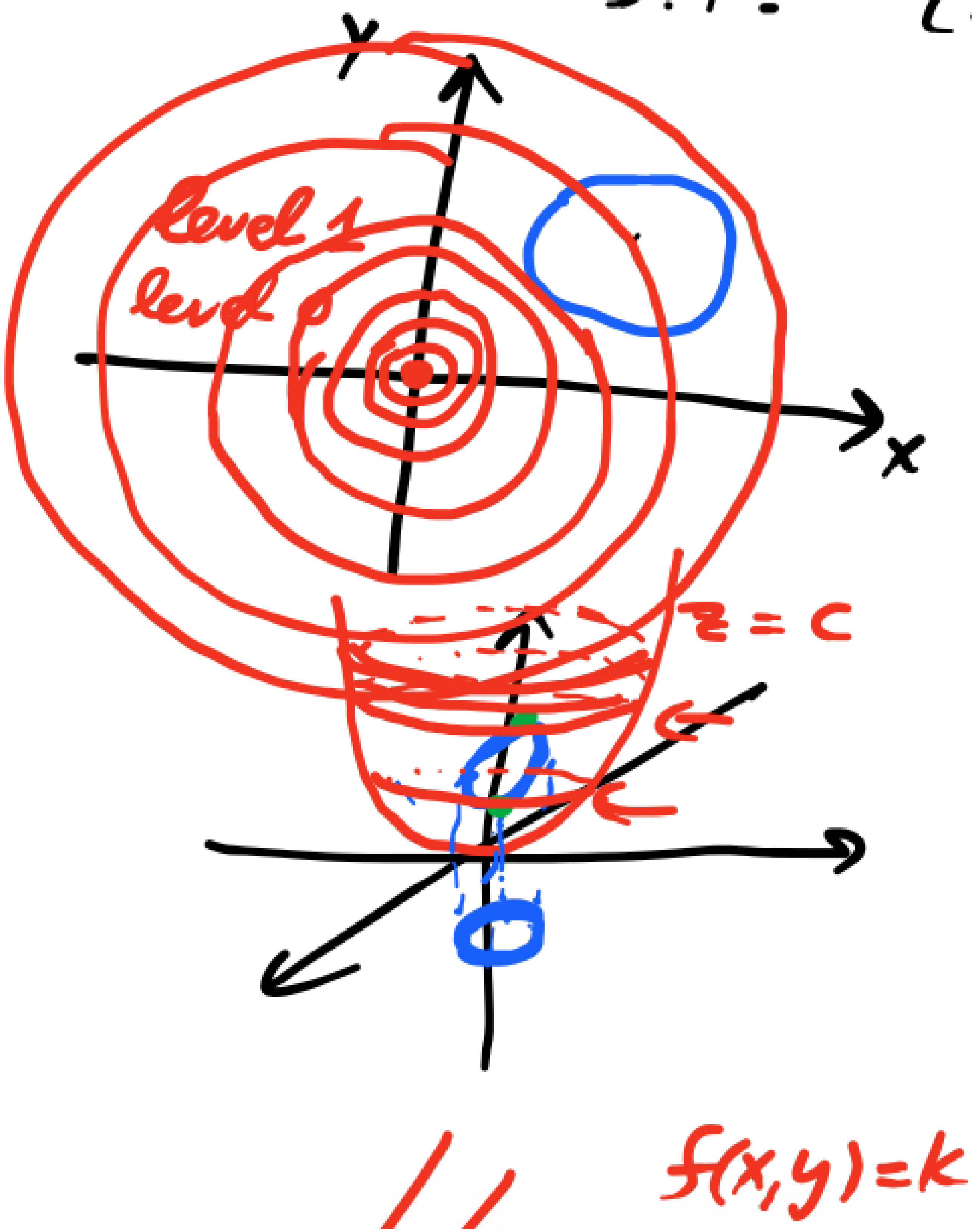
max/min $f(x,y)$

subject to $\boxed{g(x,y) = c}$

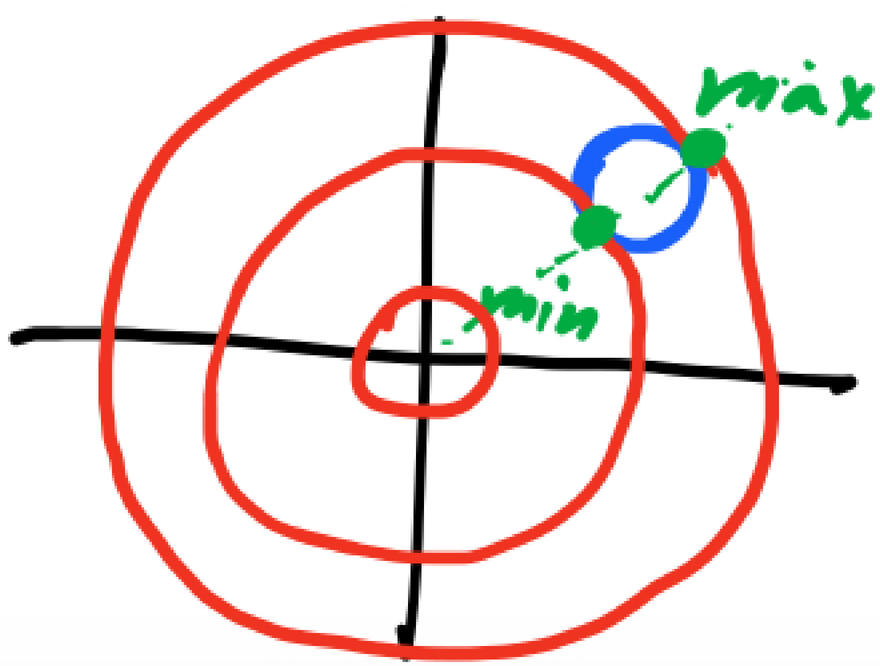
constraint.

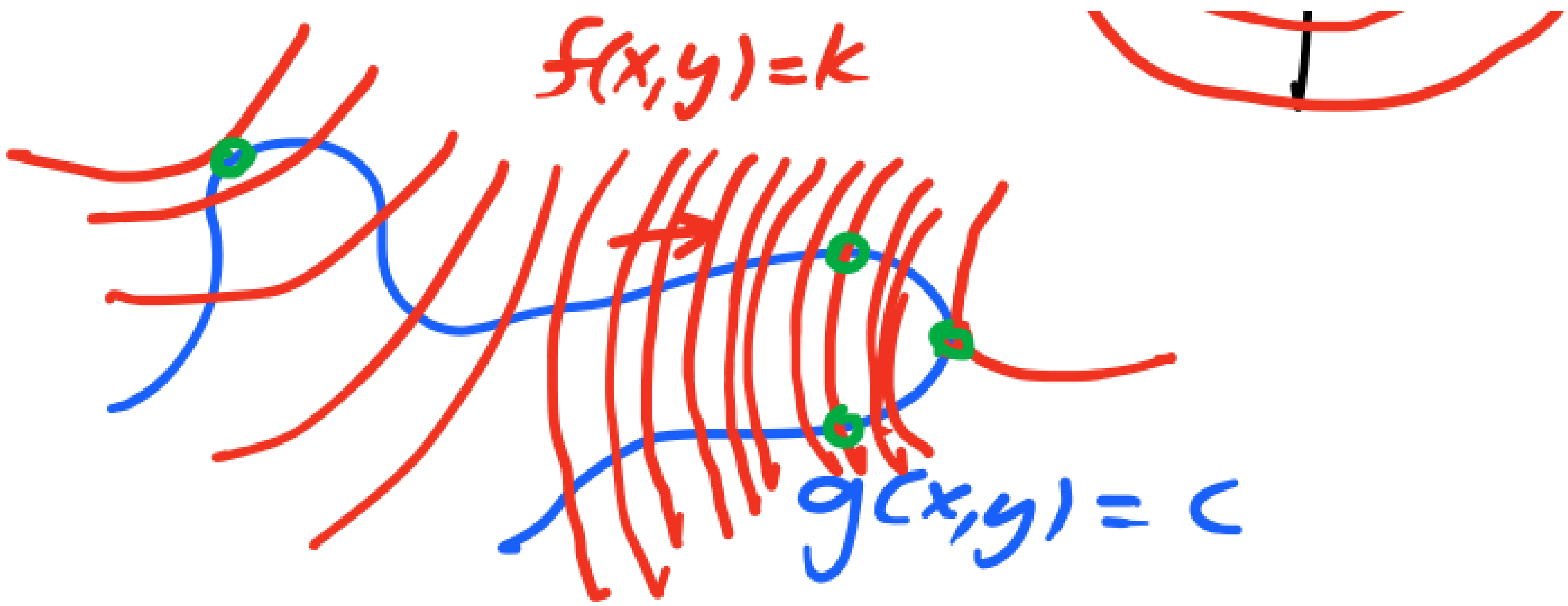
Example max/min $f(x,y) = \cancel{xy}\ x^2+y^2$

s.t. $(x-2)^2+(y-2)^2=1.$



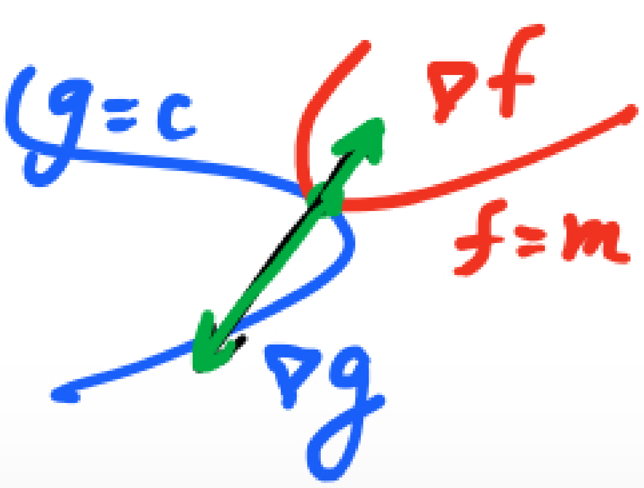
Idea look at level curves of f!





If $\nabla f \neq 0$, then at a local max/min the level curve $f(x,y)=m$ is tangent to the constraint curve $g(x,y)=c$.

$\iff$ their normal vectors are parallel!



$$\iff \boxed{\nabla f = \lambda \nabla g}$$

for some λ

Method of Lagrange multipliers

* Assumptions: - problem has solution

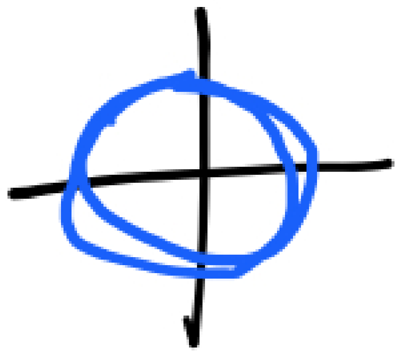
- $\nabla g \neq 0$ on $g(x,y)=c$.

- Solve the system $\begin{cases} \nabla f(x,y) = \lambda \nabla g(x,y) \\ g(x,y) = c \end{cases}$

for x, y and λ

- Evaluate f at pts (x,y) and compare

Example max/min $f(x,y)=xy$ s.t. $\underbrace{x^2+y^2}_{g(x,y)}=1$.



$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 1 \end{cases}$$

$$\nabla f = (y, x)$$

$$\nabla g = (2x, 2y)$$

$\nabla g \neq 0$ on $x^2+y^2=1$ ✓

$$\begin{cases} (x,x) = \lambda(2x, 2y) \\ x^2+y^2=1 \end{cases}$$

3 eqns, 3 variables

$$\begin{cases} y = 2\lambda x & (1) \\ x = 2\lambda y & (2) \\ x^2 + y^2 = 1 & (3) \end{cases}$$

Plug (1) into (2):

$$x = 2\lambda(2\lambda x) = 4\lambda^2 x$$

$$x(1 - 4\lambda^2) = 0$$

$$\Leftrightarrow \underline{x = 0} \quad \text{or} \quad 4\lambda^2 = 1$$

$x = 0 \Downarrow y = 0 \Downarrow x^2 + y^2 = 0$ ×

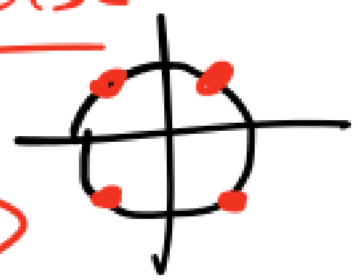
$4\lambda^2 = 1 \Downarrow \lambda = \pm \frac{1}{2} \Downarrow$

(1) $\Rightarrow$ $\underline{y = \pm x}$

(3) $\Rightarrow$ $2x^2 = 1$

$\Rightarrow \underline{x = \pm \frac{1}{\sqrt{2}}}$

Exercise



Next: Multivariable Integration!