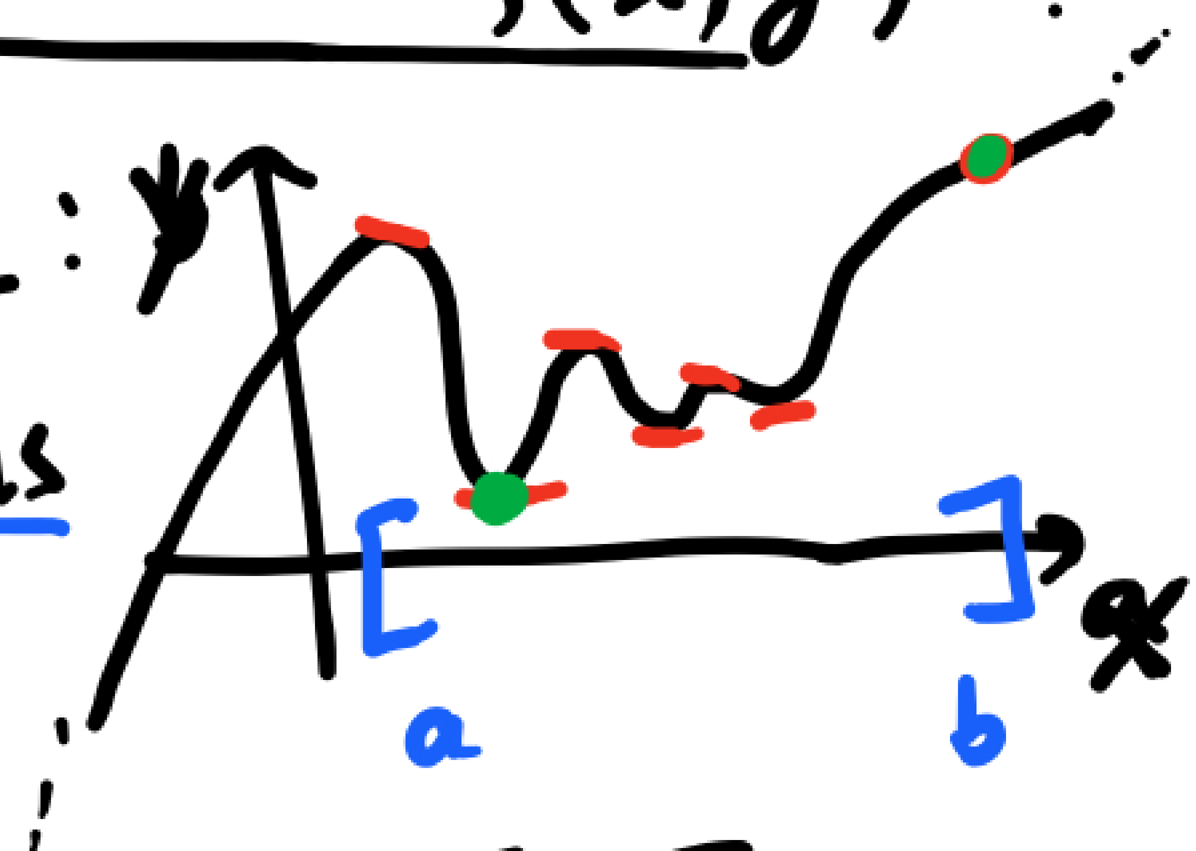
Last time:

- 2^{ND} Derivative test for $f(x,y)$
  - Locates local max/min among critical points.

↳ what about global extrema?

How to find global max and global min for f(x,y)?

Recall from Calc 1:

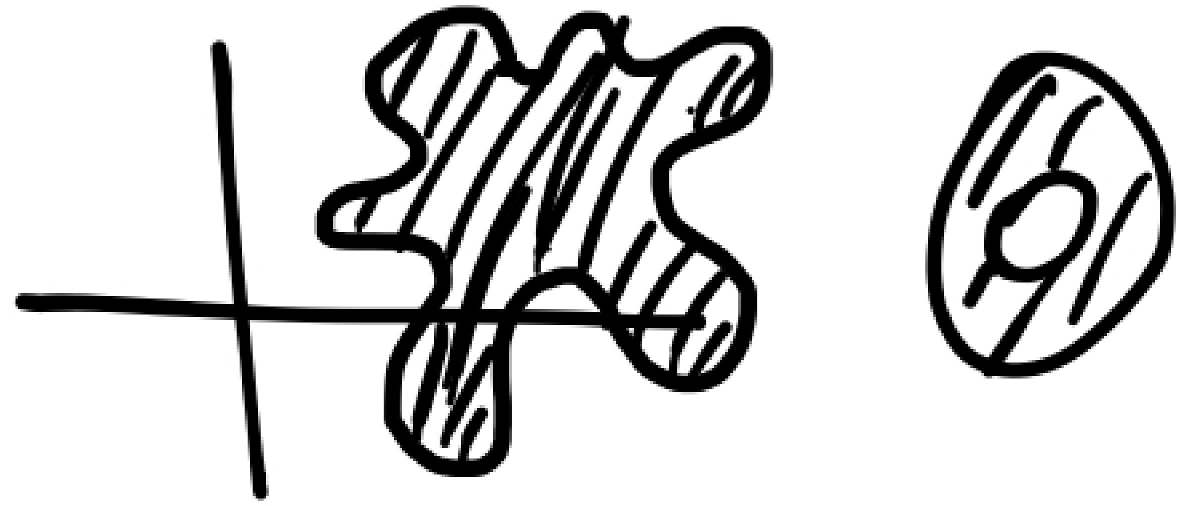


- If $f(x)$ is continuous then it has global max and global min on a closed interval $[a,b]$.

- Algorithm: ① find critical pts
② find $f(a), f(b)$.
③ compare values of ① and ②

Can we generalize it ?

$[a,b] \longrightarrow$

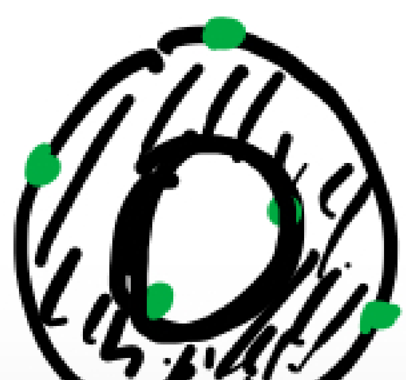


closed and bounded set in $\mathbb{R}^2$

contains all boundary pts

not closed

not bounded



closed

Extreme value theorem

- If $f(x,y)$ is continuous on a closed and bounded set K, then it attains global max and global min in K.

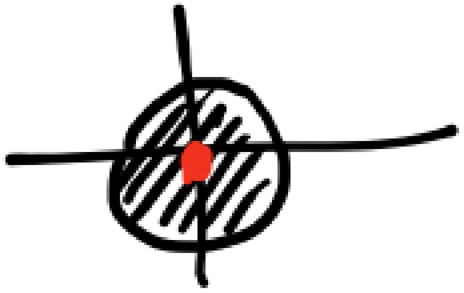
Algorithm:

1. find critical pts inside K

2. Study f on the boundary of K

3. compare

Example Find global extrema of $f(x,y) = xy$ on $D = \{(x,y) \mid x^2+y^2 \le 1\}$.

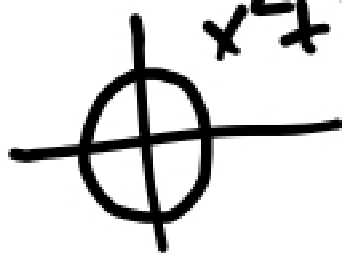


- Crit pts: $\nabla f = \vec{0}$

$\iff (y, x) = (0,0)$

$\iff$ origin.

- Boundary



- $y = \pm\sqrt{1-x^2}$

$f = \pm x\sqrt{1-x^2}$

$-1 \le x \le 1$

$\to$ Use algorithm from Calc 1

- $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$

$0 \le t \le 2\pi$

$f = \cos t \sin t$

$\to$ Use algorithm from Calc 1.

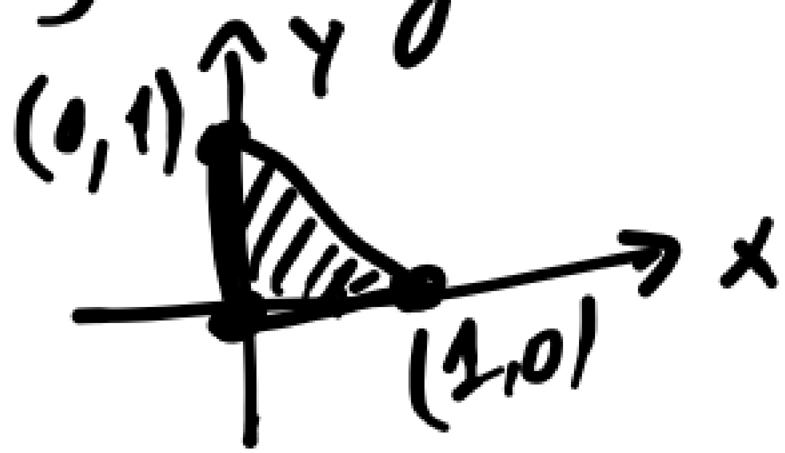
Exercise

Geometric solution:

$f = \pm$ area of rectangle

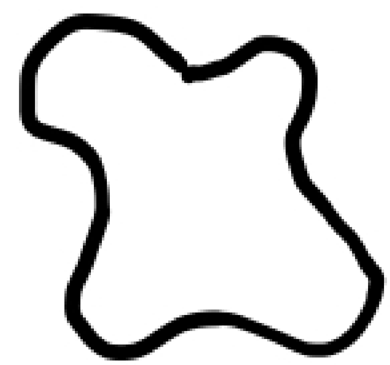
Exercise $f(x,y) = 3x+3y-x^2-y^2-xy$

on



What to do if boundary is complicated?

e.g. $x^4+xy+y^4=1$



$x=\,??$

$y=\,??$

Answer: **Lagrange multipliers**

Problem

max/min $f(x,y)$ → Constrained optimization

subject to $\boxed{g(x,y)=c}$

"constraint"