## Last time:

**Thm** If $f(x,y)$ is differentiable at $(a,b)$, then its directional derivatives at $(a,b)$ exists in all directions $\hat{u}$ and moreover

$$\boxed{D_{\hat{u}} f(a,b) = \nabla f(a,b) \cdot \hat{u}}$$

**Note** Same works for more variables.

**Example** $f(x,y,z) = xy + ye^{z^2}$, find

$$D_{\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)} f(1,2,3).$$

$$\nabla f(x,y,z) = (f_x, f_y, f_z) = (y, x + e^{z^2}, 2yze^{z^2})$$

$$\nabla f(1,2,3) = (2, 1 + e^9, 12e^9).$$

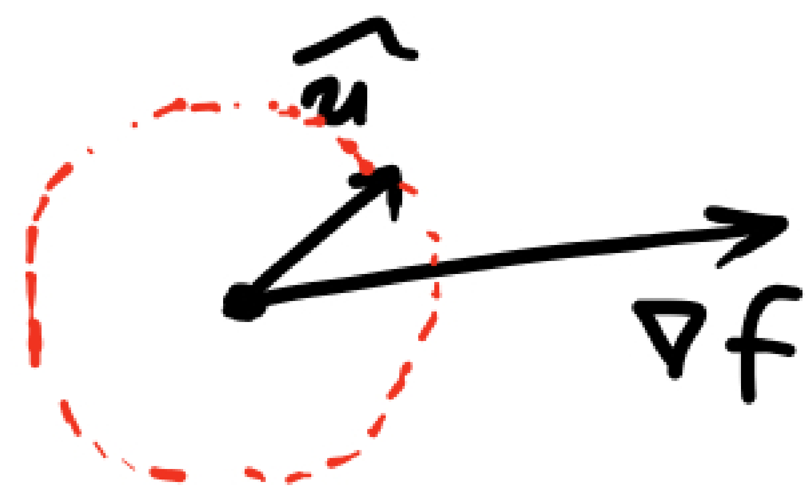
$$D_{\hat{u}} f(1,2,3) = (2, 1 + e^9, 12e^9) \cdot \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right) =$$

$$= \sqrt{3} - \frac{11}{\sqrt{3}} e^9.$$

# Significance of $\nabla f$:

(a) Observation: $$D_{\hat{u}} f = \nabla f \cdot \hat{u}$$

$$= \|\nabla f\| \underbrace{\|\hat{u}\|}_{=1} \overbrace{\cos\theta}^{-1 \le \quad \le 1}$$

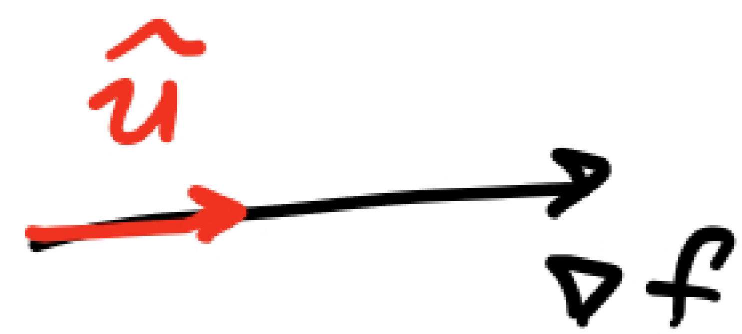



$$\Rightarrow -\|\nabla f\| \le D_{\hat{u}} f \le \|\nabla f\|$$

when they are equal?

$\cos\theta = -1$ and $1$

$\theta = \pi$ and $0$




- The value of $D_{\hat{u}} f$ is maximized when $\hat{u}$ points in the direction of $\nabla f$

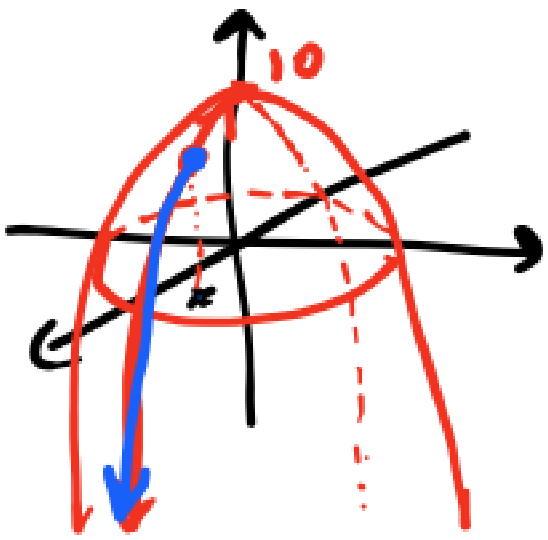
max value $= \|\nabla f\|$

- The value of $D_{\hat{u}} f$ is minimized when $\hat{u}$ points in the opposite direction to $\nabla f$.

min value $= -\|\nabla f\|$

Example $f(x,y) = 10 - (x^2+y^2)$

$(x,y) = (2,1)$




$\nabla f(x,y) = (f_x, f_y) =$

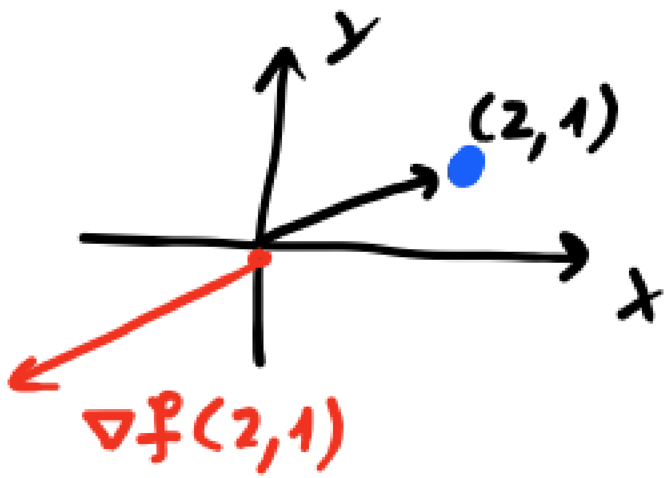
$= (-2x, -2y)$

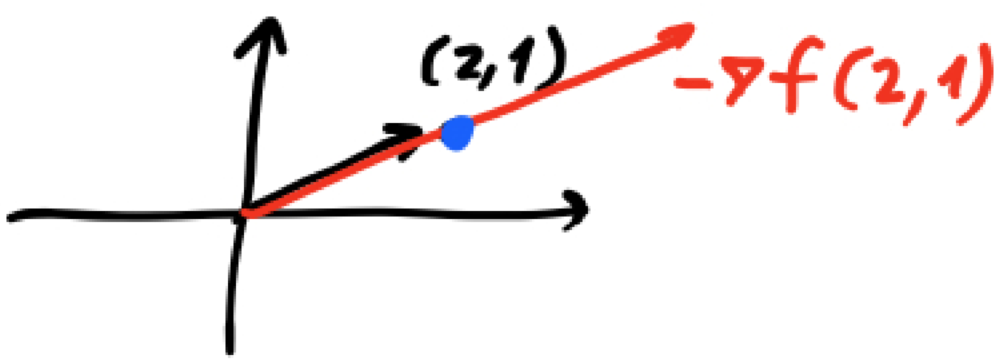
$\nabla f(2,1) =$

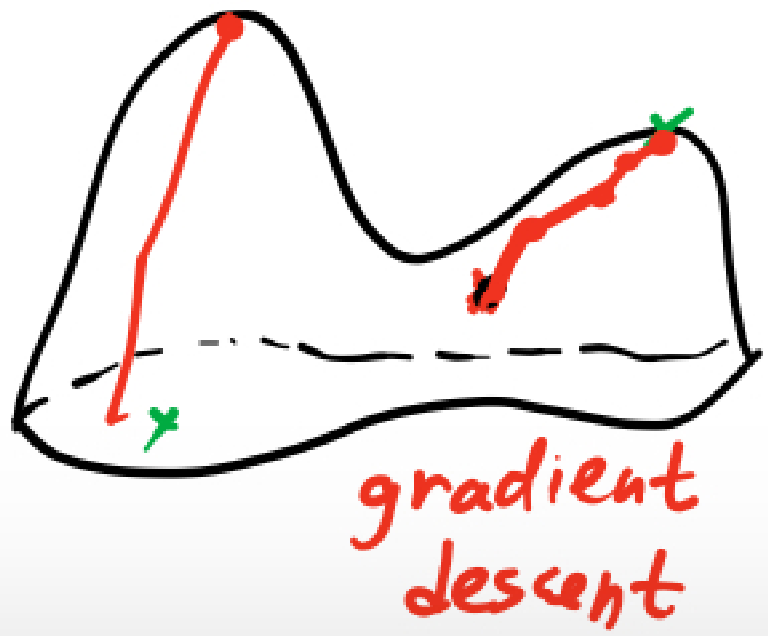
$= (-4, -2) =$

$= (-2) \cdot (2,1).$

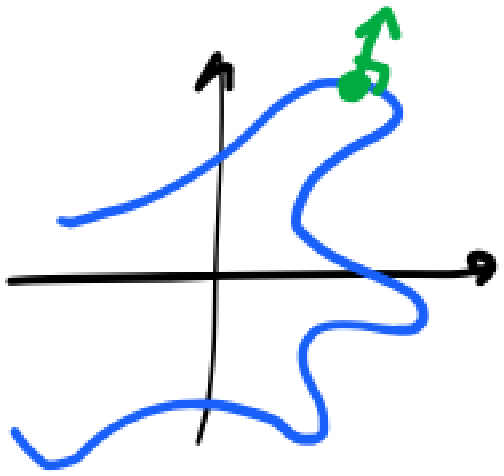
times



(b) "$\nabla f$ gives normal vectors to level curves and level surfaces"



$$f(x,y) = c$$