Last time:

Clairaut's theorem:

If f_{xy} and f_{yx} exist near (a,b) and continuous at (a,b), then

$$\underline{f_{xy}}(a,b) = \underline{f_{yx}}(a,b).$$

Example when $f_{xy} \neq f_{yx}$

$$f(x,y) = \begin{cases} xy\dfrac{x^2-y^2}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

More notation: If $f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f$.

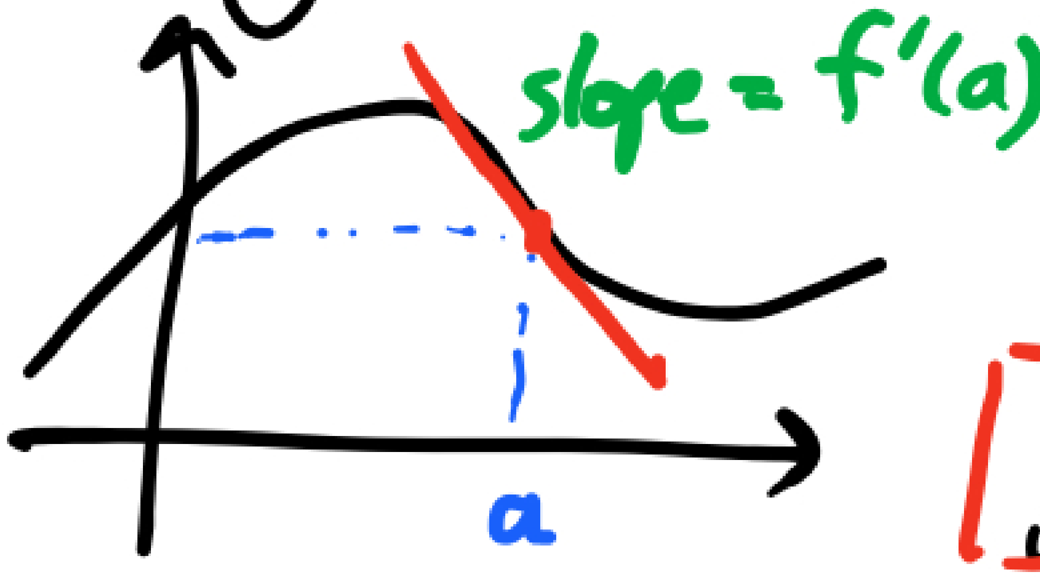
then $\underline{f_{xy}} = (f_x)_y = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) =$

$$= \frac{\partial^2 f}{\partial y \, \partial x}.$$

Similarly, $f_{yx} = \dfrac{\partial^2 f}{\partial x \, \partial y}$.

14.4. Tangent Planes & Linear Approximations

Calc 1



$y = f(x)$

$\boxed{f(x) \approx \text{tangent line}}$

when $x \approx a$

eq^n:
at $(a, f(a))$

$$y - f(a) = f'(a)(x-a)$$

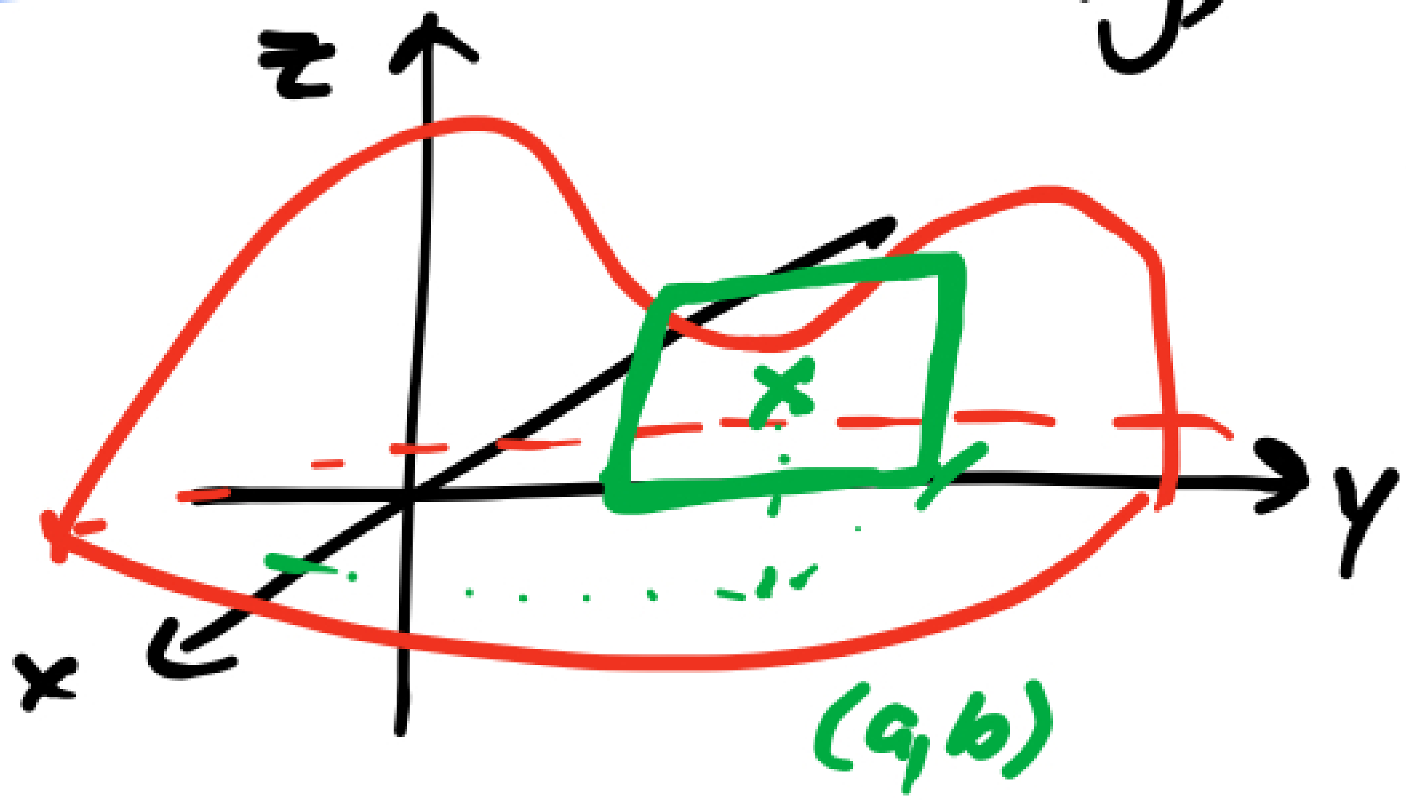
$$\Rightarrow y = f(a) + f'(a)(x-a)$$

$$f(x) \approx f(a) + f'(a)(x-a)$$

when $x \approx a$

best linear approx. of f near a

Generalize to $f(x,y)$!

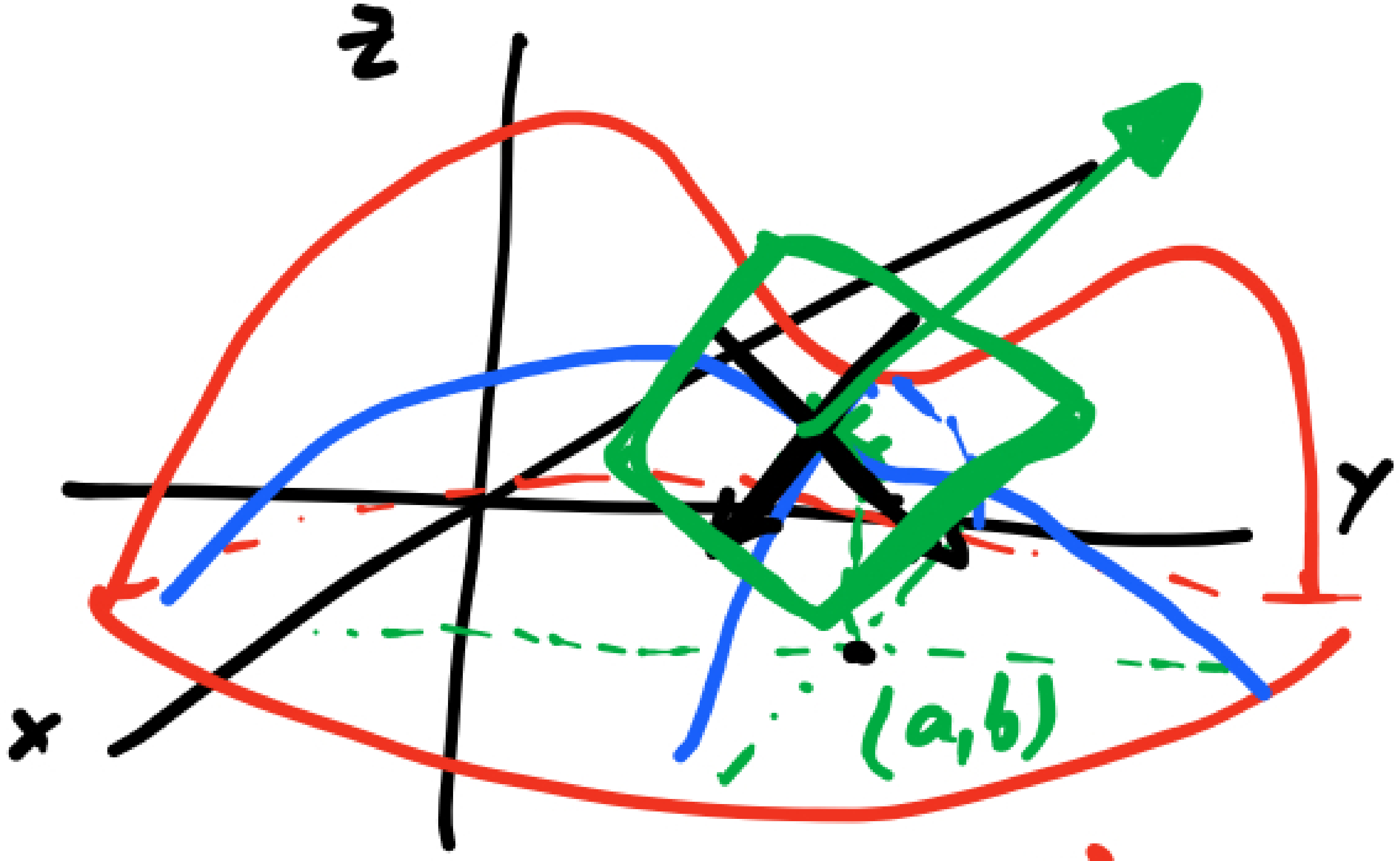


$z = f(x,y)$

Eq^{n}:

$\boxed{\text{tangent plane to surface } z=f(x,y) \text{ at } (a,b,f(a,b)).}$

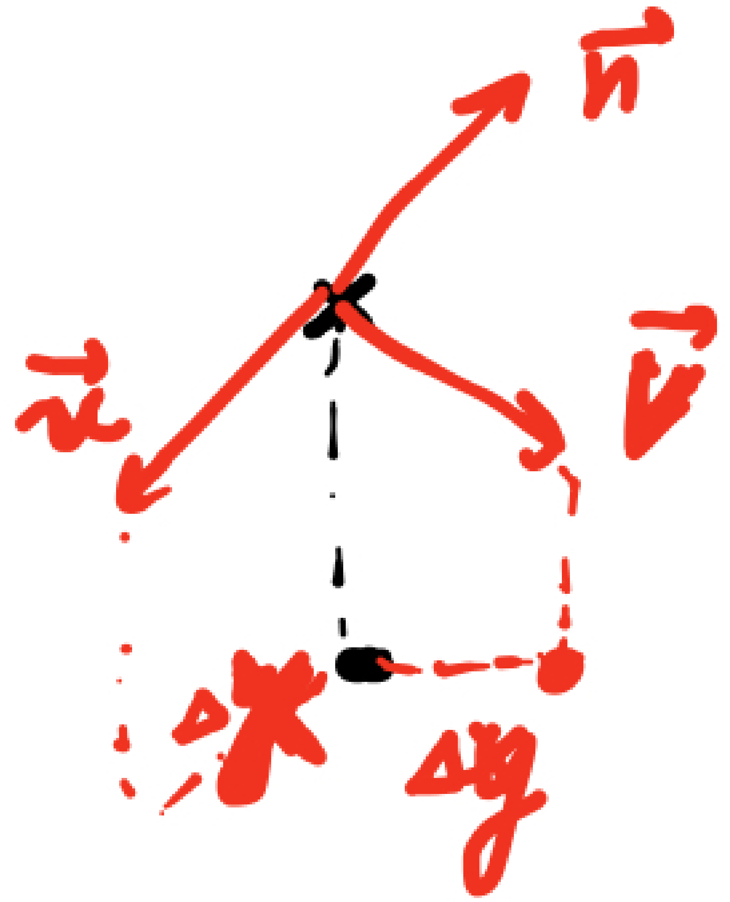
- need a point ✓
- need a normal direction ✓



$$\vec{n} = \vec{u} \times \vec{v}$$

$$\vec{u} = (\Delta x, 0, f_x(a,b)\Delta x)$$

$$\vec{v} = (0, \Delta y, f_y(a,b)\Delta y)$$



$$\vec{n} = \vec{u} \times \vec{v} = \underbrace{(-f_x(a,b), -f_y(a,b), 1)}_{\cdot \Delta x \Delta y}$$

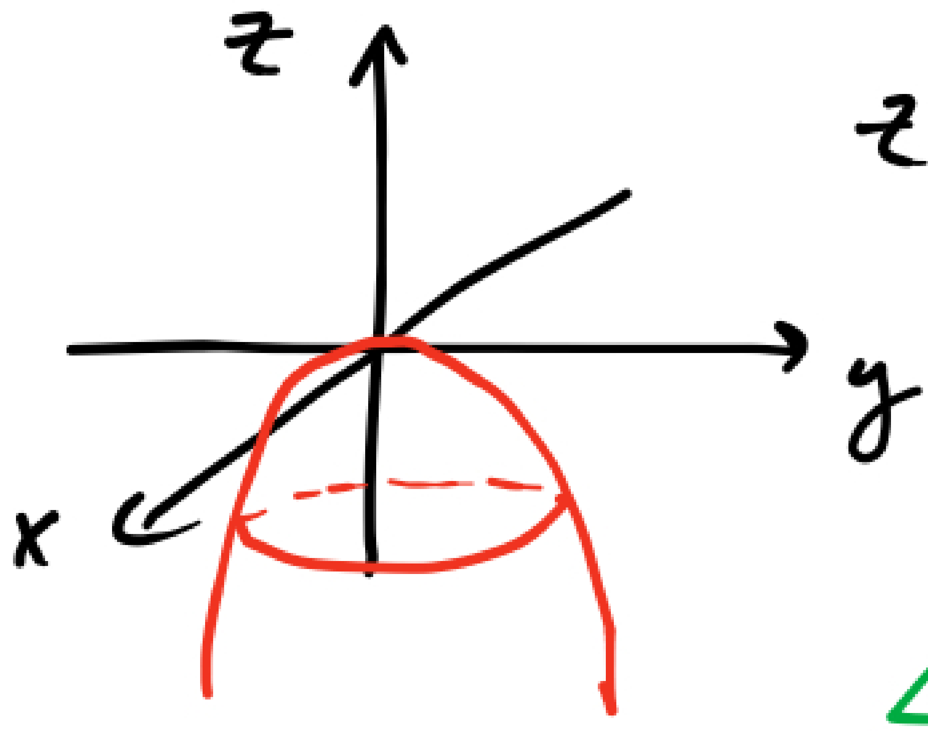
$\underline{Eq^n}$: $-f_x(a,b)(x-a) - f_y(a,b)(y-b) + (z - f(a,b)) = 0$

$$\boxed{z = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)}$$

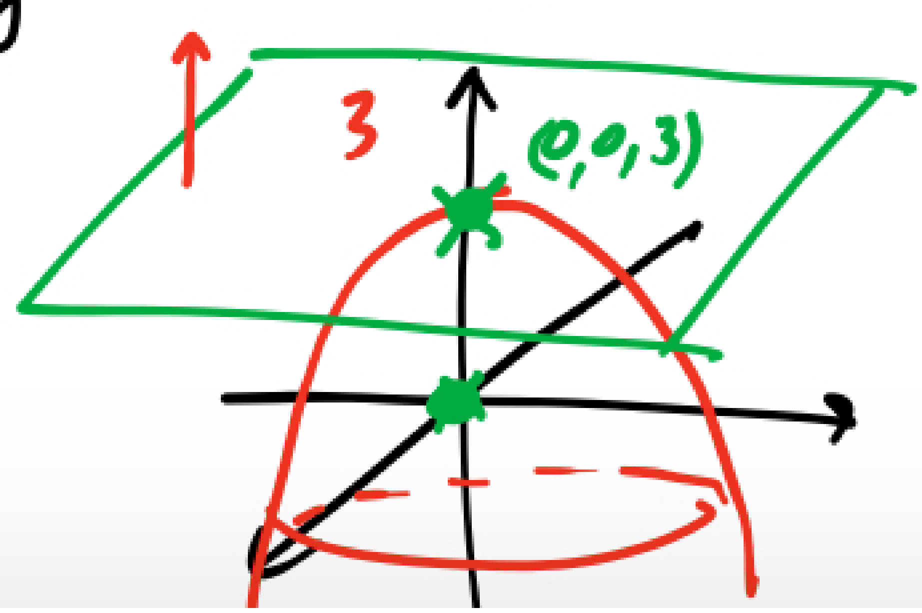
compare with $y = f(a) + f'(a)(x-a)$!

Example $f(x,y) = 3 - x^2 - y^2$

· find tangent plane for $(x,y) = (0,0)$



$z = -x^2 - y^2 =$
$= -(x^2 + y^2)$



$z = 3 - x^2 - y^2$

Verify: $f_x = -2x \qquad f_x(0,0) = 0$

$f_y = -2y \qquad f_y(0,0) = 0$

$$z = 3$$

Ex:

tangent plane at $(1,1,1)$?