

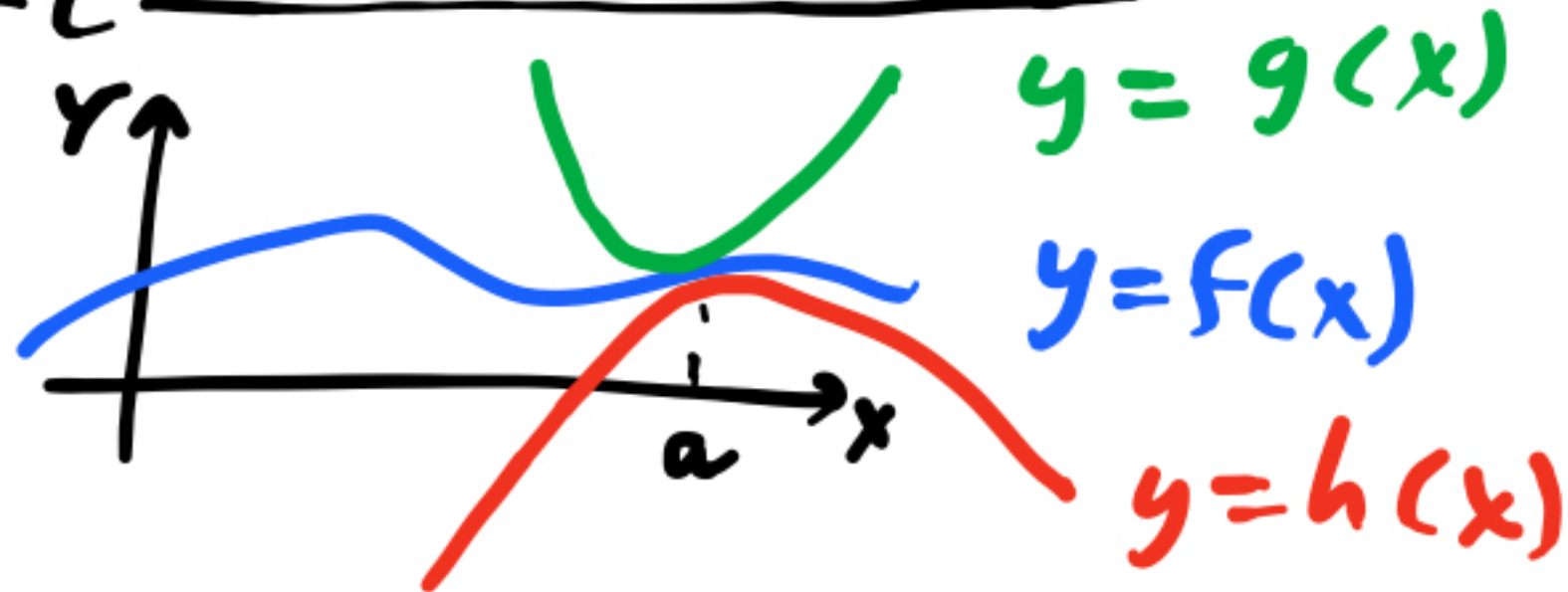
---

⑤  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2} =$

Try  $y = mx$ ,  $y = mx^2$ ,  $x = my^2$   
... get 0.

Limit is zero!  
~~...~~

# Squeeze theorem:



If  $h(x, y) \leq f(x, y) \leq g(x, y)$   
for all  $(x, y)$  near  $(a, b)$ .

and if  $\lim_{(x, y) \rightarrow (a, b)} h(x, y) = \lim_{(x, y) \rightarrow (a, b)} g(x, y) = L$

then  $\lim_{(x, y) \rightarrow (a, b)} f(x, y) = L$  . too .

$$\frac{x^2 y^2}{x^2 + y^2}$$

Note

$$0 \leq \frac{y^2}{x^2 + y^2} \leq 1$$

where  
defined

since  $y^2 \leq x^2 + y^2$

Then

$$x^2 \cdot 0 \leq \frac{x^2 y^2}{x^2 + y^2} \leq x^2 \cdot 1$$

$= 0$

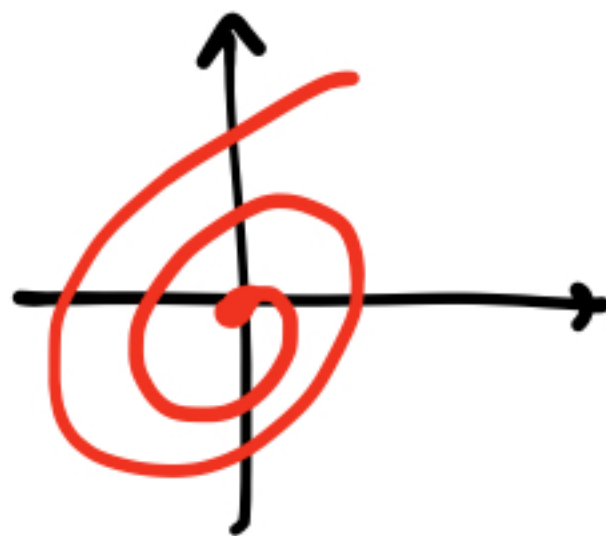
Squeezed  
by functions that  $\rightarrow 0$   
as  $(x, y) \rightarrow 0$ . Thus  $\lim_{(x, y) \rightarrow (0, 0)} \frac{x^2 y^2}{x^2 + y^2} = 0$

$$(6) \lim_{(x,y) \rightarrow (0,0)} \frac{\sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}$$

• try polar coordinates.

$$\frac{\sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \frac{\sin r}{r}$$

As  $(x,y) \rightarrow (0,0)$ ,  $r \rightarrow 0^+$   
and vice versa!



$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin \sqrt{x^2+y^2}}{\sqrt{x^2+y^2}} = \lim_{r \rightarrow 0^+} \frac{\sin r}{r} = 1.$$

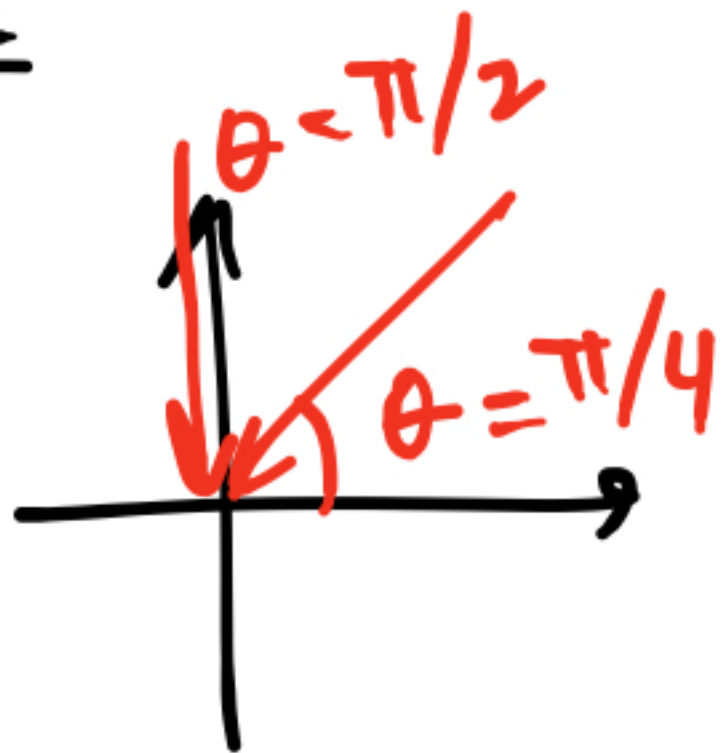
from  
Calc 1

$$\textcircled{7} \lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2+y^2} = \lim_{r \rightarrow 0^+} \frac{(r \cos \theta)^2}{r^2} =$$

$$= \lim_{r \rightarrow 0^+} \cos^2 \theta$$

depends on  $\theta$ !

DNE





⑧

$\lim_{(x,y) \rightarrow (1,2)}$

$$\frac{x^2 + y^2}{x - y}$$

$$= \frac{1^2 + 2^2}{1 - 2} = \boxed{-5}$$

Since

$\frac{x^2 + y^2}{x - y}$  is continuous  
when  $x \neq y$

Fact: polynomial and rational functions  
are continuous where defined  
(when denominator  $\neq 0$ ).

Thus  $\lim_{(x,y) \rightarrow (a,b)} f(x,y) = f(a,b)$ .

Exercise :  $\lim_{(x,y) \rightarrow (1,1)} \frac{x^2 + y^2}{x - y}$ .

## 14.3. Partial Derivatives

Motivation:  $f(x, y) \leftarrow$  How does it change when  $x$  and  $y$  change!

(compare:  $f(x)$  - rate of change is controlled by derivative)

Question: How to define "derivative" for multivar function?