

14.1. Functions of several variables.

$$f(x) \quad f(x, y) \quad f(x, y, z) \quad \dots \quad f(x_1, \dots, x_n)$$

Graph of $y = f(x)$:

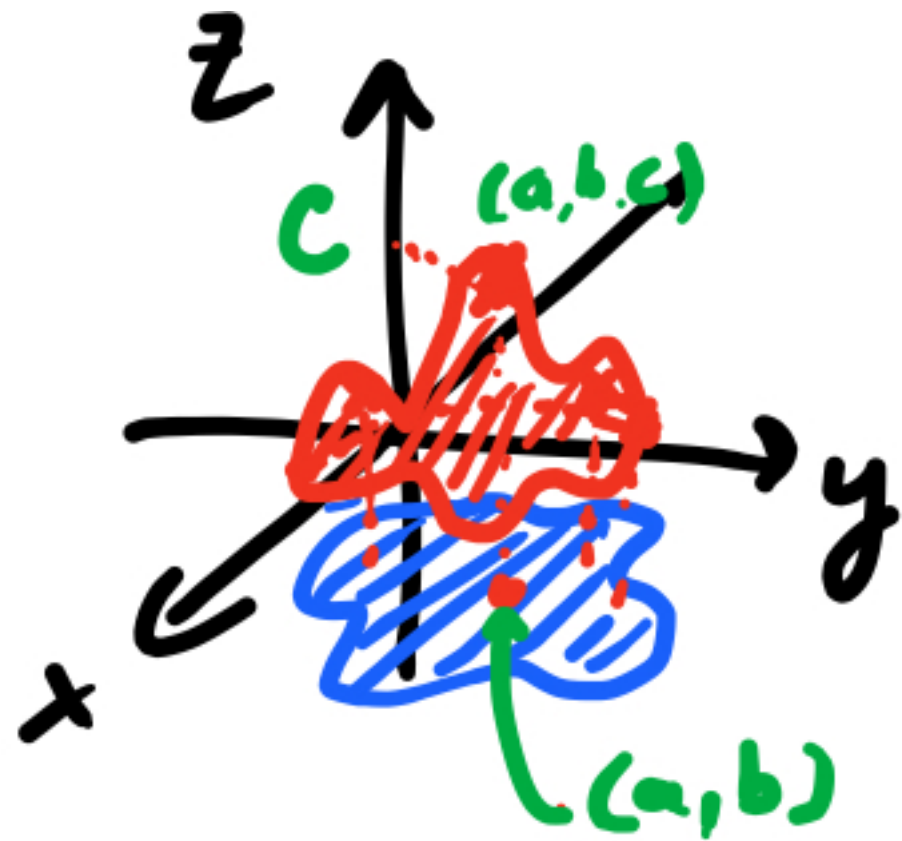
$$x \in D \subseteq \mathbb{R}$$

domain



Graph of $z = f(x, y)$:

$$(x, y) \in D \subseteq \mathbb{R}^2$$

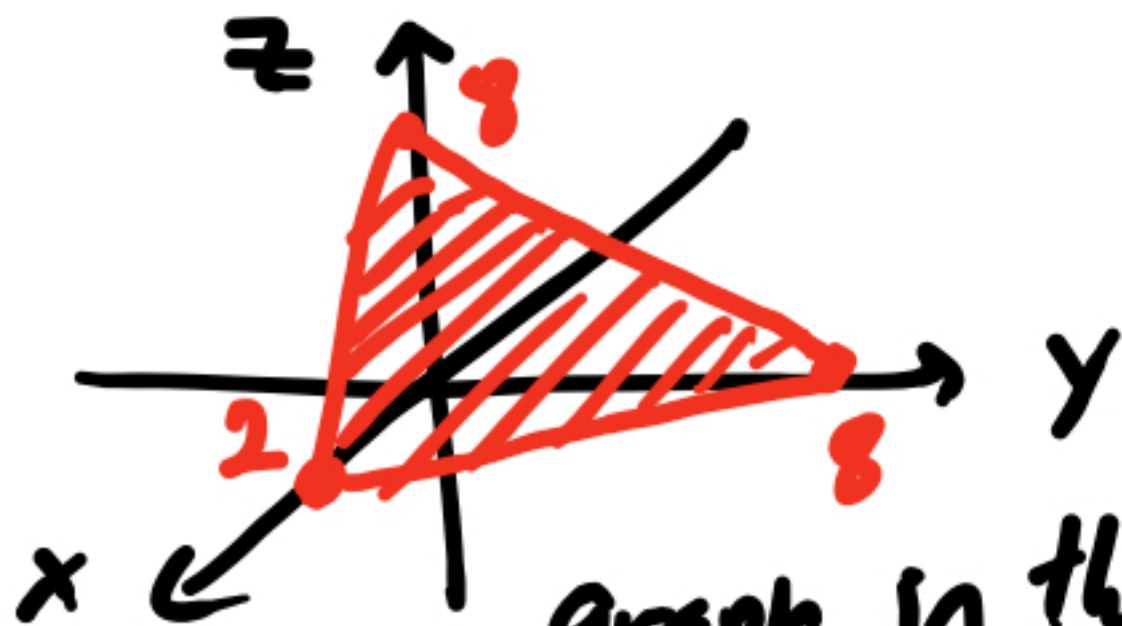


$$f(a, b) = c$$

Examples ① $z = 8 - 4x - y$

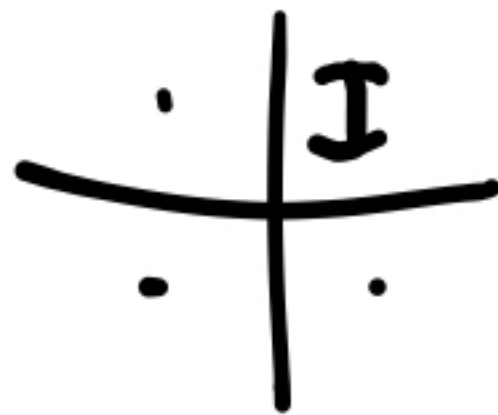
$$4x + y + z - 8 = 0$$

a plane



graph in the
first octant
($x, y, z \geq 0$)

$$4x + y + z = 8$$



②

$$z = \sqrt{4 - x^2 - y^2} \Leftrightarrow z^2 = 4 - x^2 - y^2$$

$$x^2 + y^2 + z^2 = 4.$$

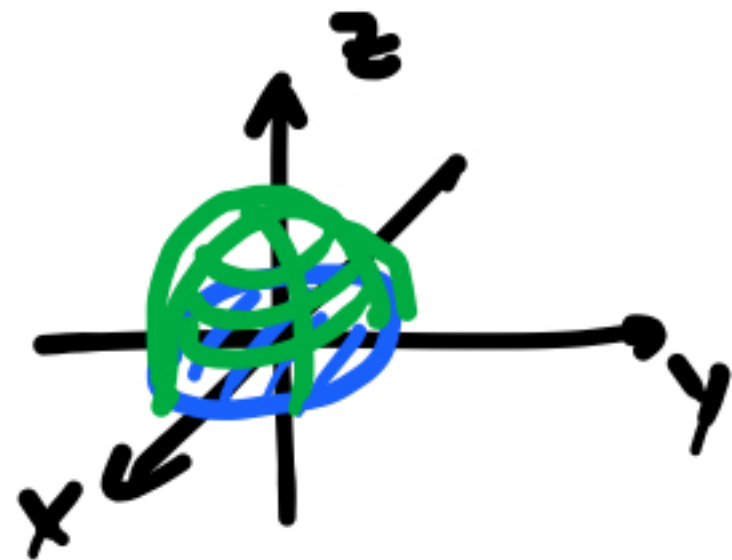
domain: $4 - x^2 - y^2 \geq 0$

$$z \geq 0$$

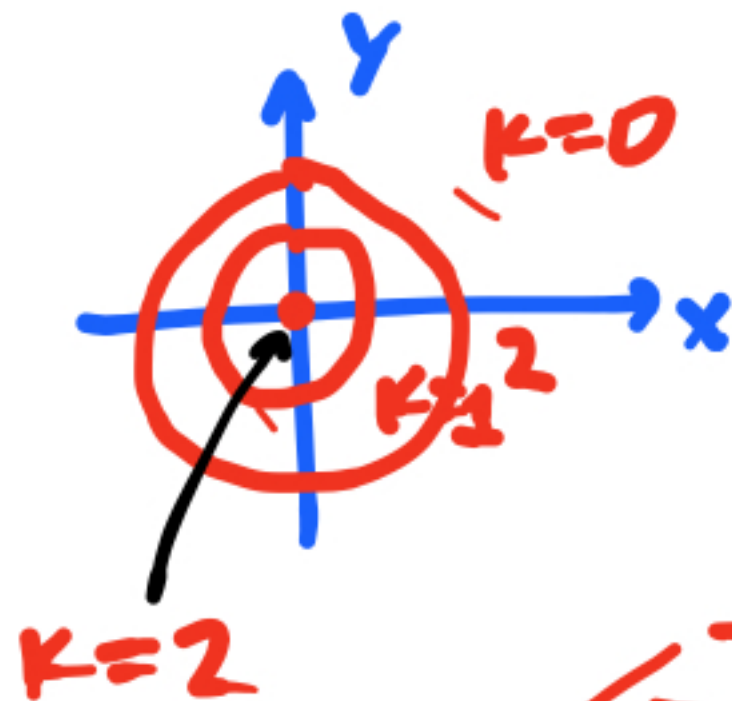
$$x^2 + y^2 \leq 4.$$



upper
hemi-sphere



Level curves: $z = k$ constant



$$k = \sqrt{4 - x^2 - y^2}$$

$$\Rightarrow x^2 + y^2 = 4 - k^2 \quad k \geq 0$$



③ $z = \sqrt{x^2 + y^2} \Leftrightarrow z^2 = x^2 + y^2$

domain: \mathbb{R}^2

(double) cone
 $z \geq 0$

$z = k$

$k = \sqrt{x^2 + y^2}$

$x^2 + y^2 = k^2$

circle of radius k

