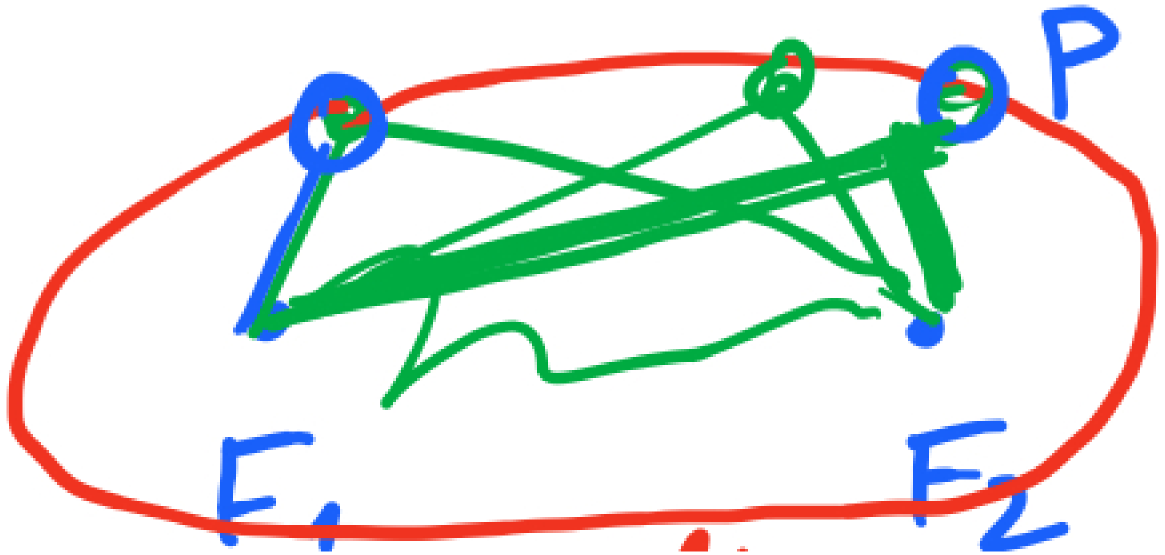
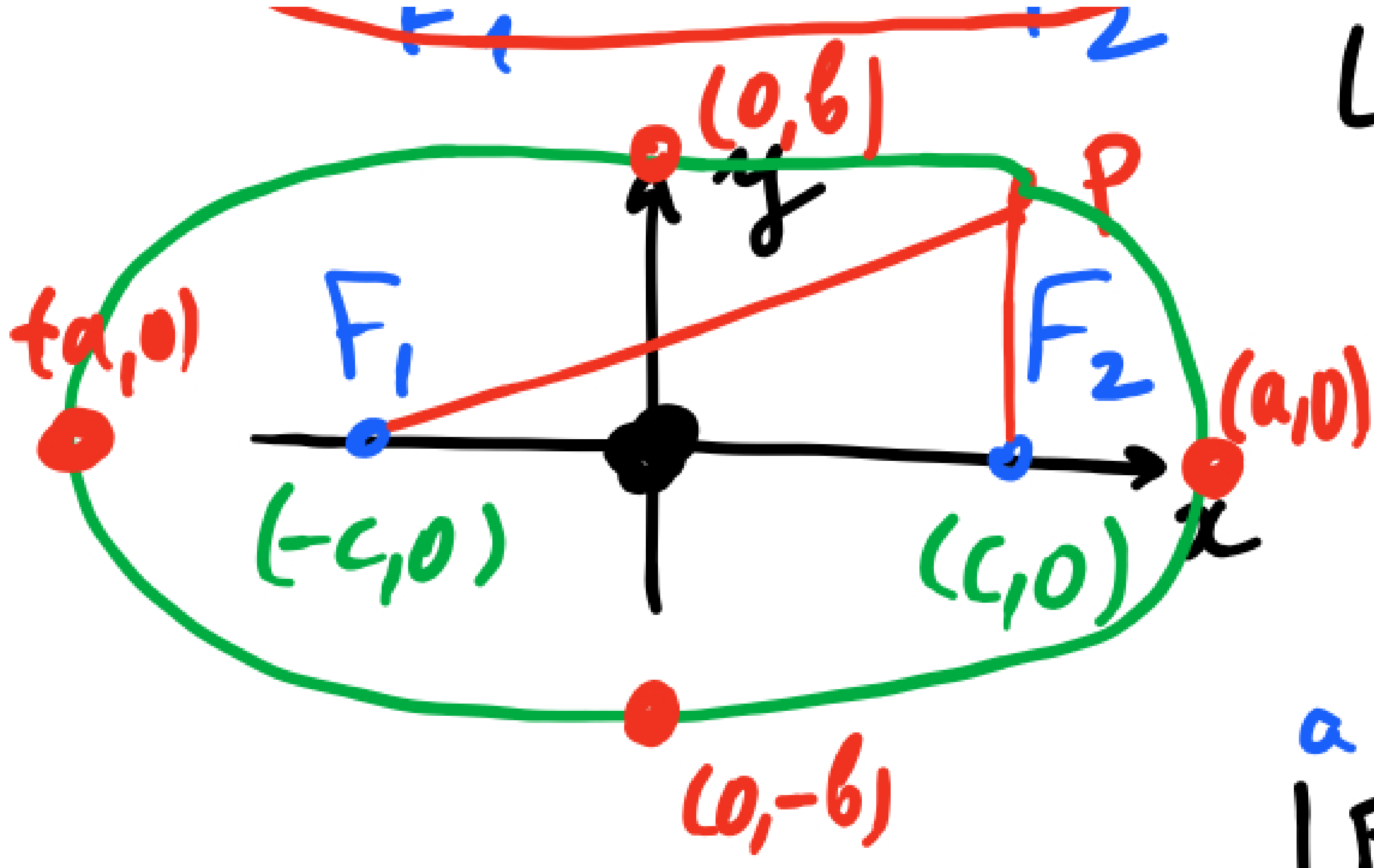
Conic sections

Ellipses

Def: an ellipse is the set of all pts on the plane with a constant $(=k)$ sum of distances from point F_1 and F_2 (foci)



Equation:



Let $F_1=(-c,0)$, $F_2=(c,0)$ and sum of distances $=2a>0$ $(a>c)$.

If $P=(x,y)$ is on ellipse, then:

a 2a a 0

$$|PF_1|+|PF_2|=2a$$

$$\sqrt{(x+c)^2+y^2}+\sqrt{(x-c)^2+y^2}=2a$$

⋮ simplify (check!)

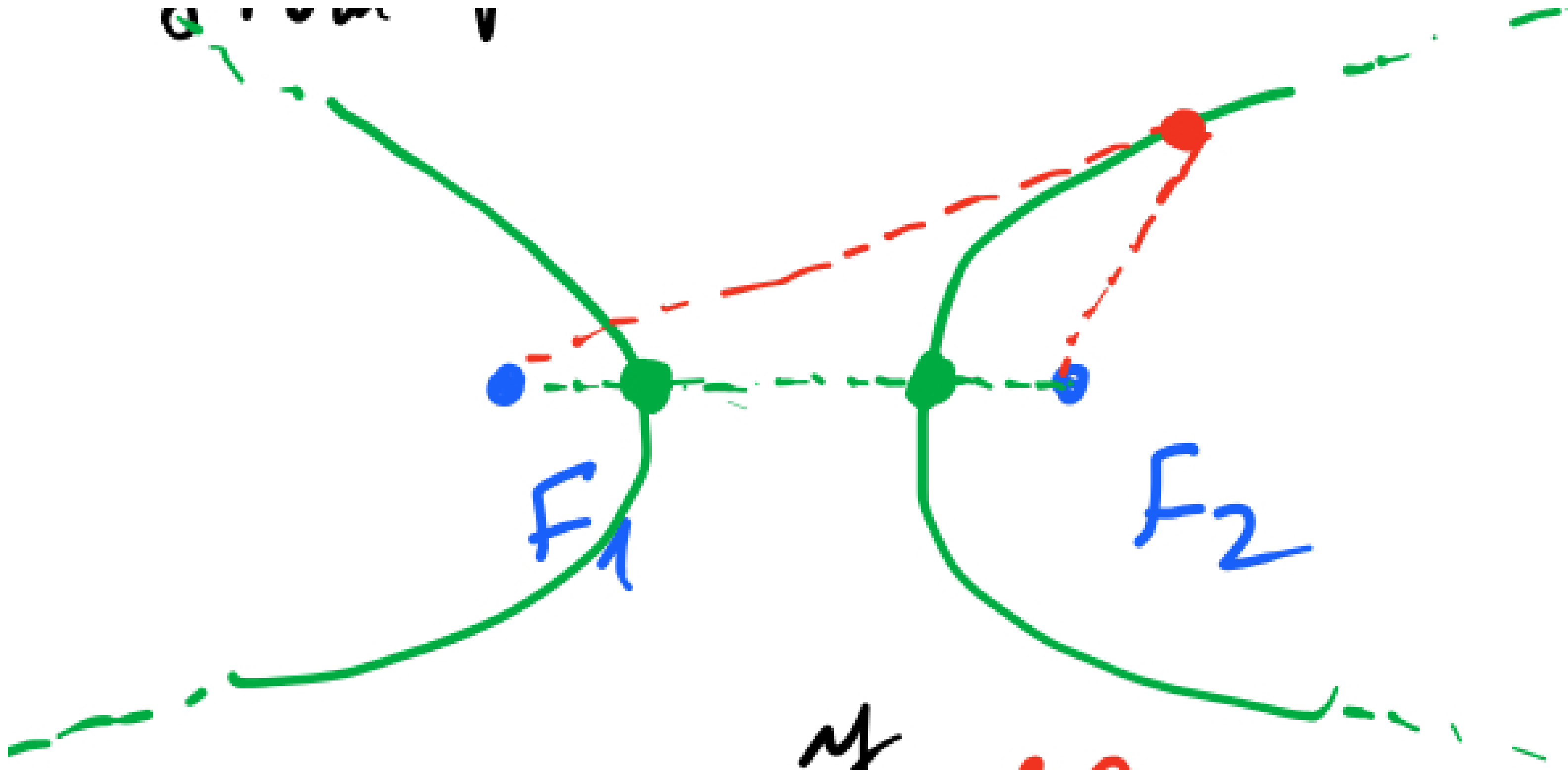
$$\boxed{\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad b^2=a^2-c^2}$$

equation of ellipse with foci $(-c,0),(c,0)$

and vertices $(-a,0)$
$(a,0)$.

Hyperbola

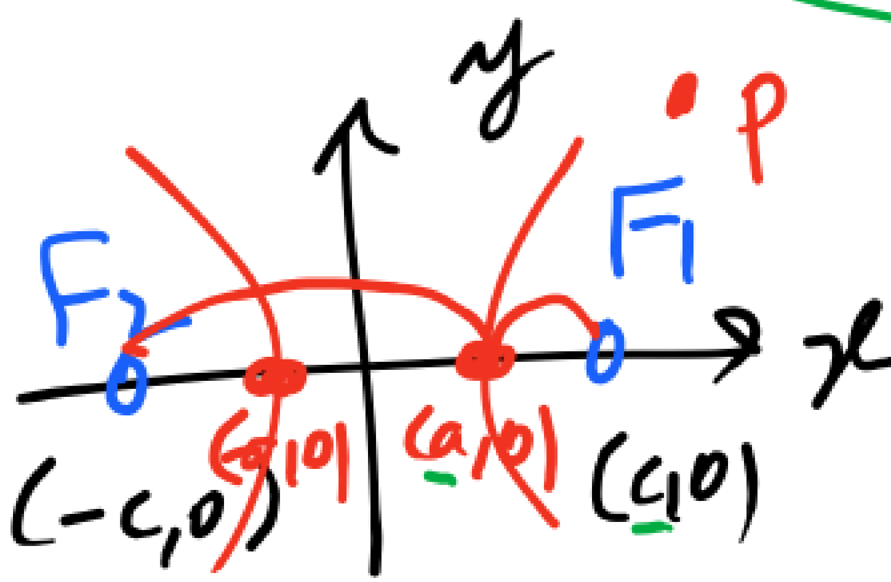
Def: a hyperbola is the set of all pts on the plane with a constant $(=k)$ (absolute value) of difference of distances from pts F_1 and F_2 (foci).



Equation!

Let $F_1=(-c,0)$

$F_2=(c,0)$



If $P=(x,y)$ is on hyperbola, then

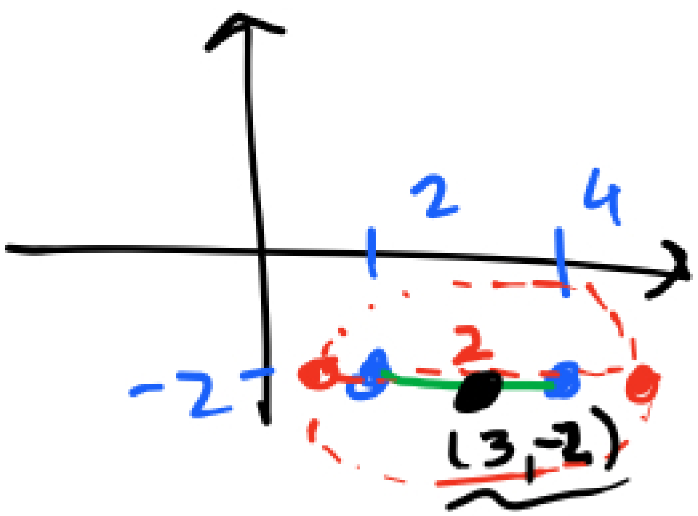
$$|PF_1|-|PF_2|=\pm 2a \quad (a<c)$$

$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad b^2=c^2-a^2}$$

equation of hyperbola

Shifted conics

Example Find an equation of the ellipse with foci $(2,-2), (4,-2)$ and vertices $(1,-2), (5,-2)$.



We have: $c = 1$

$a = 2$

Then $a^2 = 4$.

$b^2 = a^2 - c^2 = 4 - 1 = 3$.

$$\frac{(x-3)^2}{4} + \frac{(y+2)^2}{3} = 1.$$

Example Sketch the conic

$\rightarrow 9x^2 - 4y^2 - 72x + 8y + 176 = 0.$

Complete the squares

$(\quad)^2 \pm (\quad)^2 = \text{const}$

$(9x^2 - 72x) - (4y^2 - 8y) = -176$

$(4y^2 - 8y) - (9x^2 - 72x) = 176$

$4(y^2 - 2y) - 9(x^2 - 8x) = 176$

$$4(y^2-2y+1)-9(x^2-8x+16)$$
$$=176$$
$$+4-9\cdot16$$

$$4(y-1)^2-9(x-4)^2=36.$$

$$\frac{(y-1)^2}{9}-\frac{(x-4)^2}{4}=1.$$

shifted hyperbola.