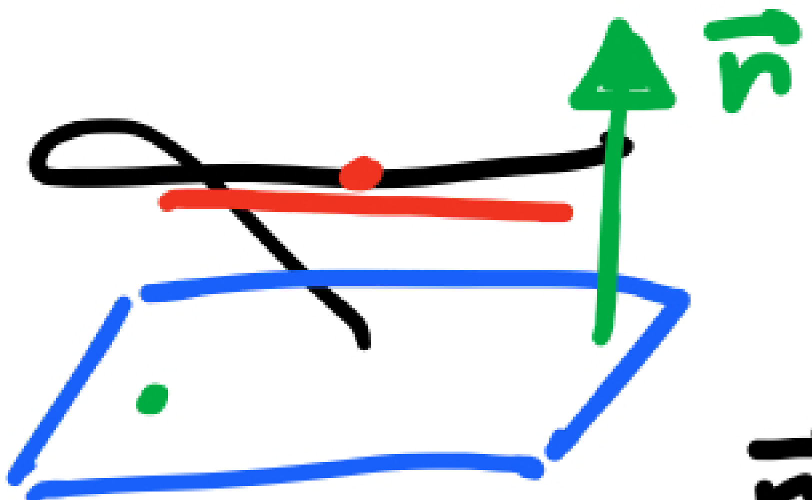
Example Is there a point on twisted cubic $\vec{r}(t) = (t, t^2, t^3)$ where the tangent line is parallel to the plane $z = 1 - 2x$?



$z = 1 - 2x \iff 2x + z - 1 = 0$

$\vec{n} = (2, 0, 1)$

$\vec{r}'(t) = (1, 2t, 3t^2)$

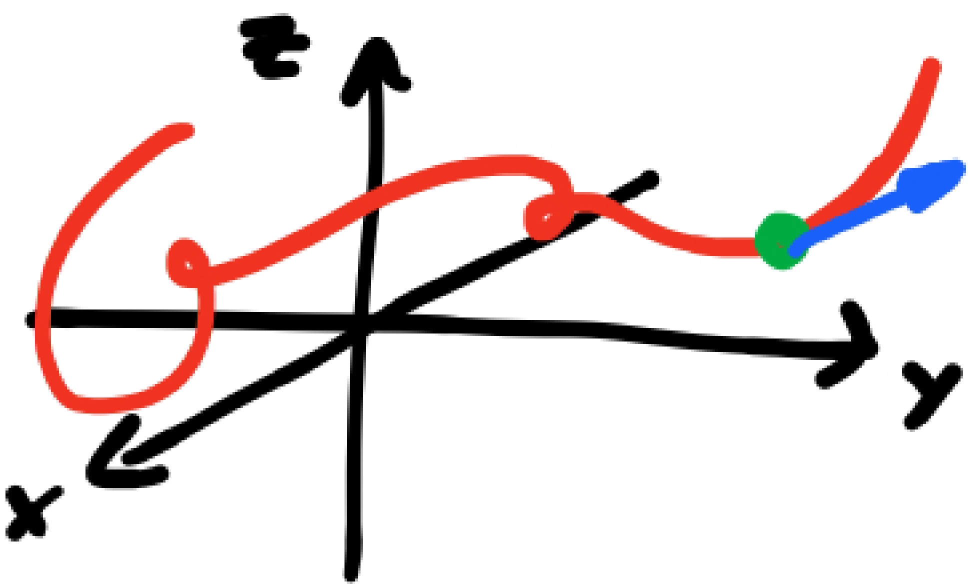
- direction vector of the tangent line at time t

$\vec{r}'(t) \cdot \vec{n} = 2 + 2t \cdot 0 + 3t^2 \cdot 1 = 2 + 3t^2$

- never zero

NO.

Space curves: velocity & acceleration



- $\vec{v}(t_0) = \lim_{\Delta t \to 0} \dfrac{\vec{r}(t_0+\Delta t) - \vec{r}(t_0)}{\Delta t}$

  velocity vector

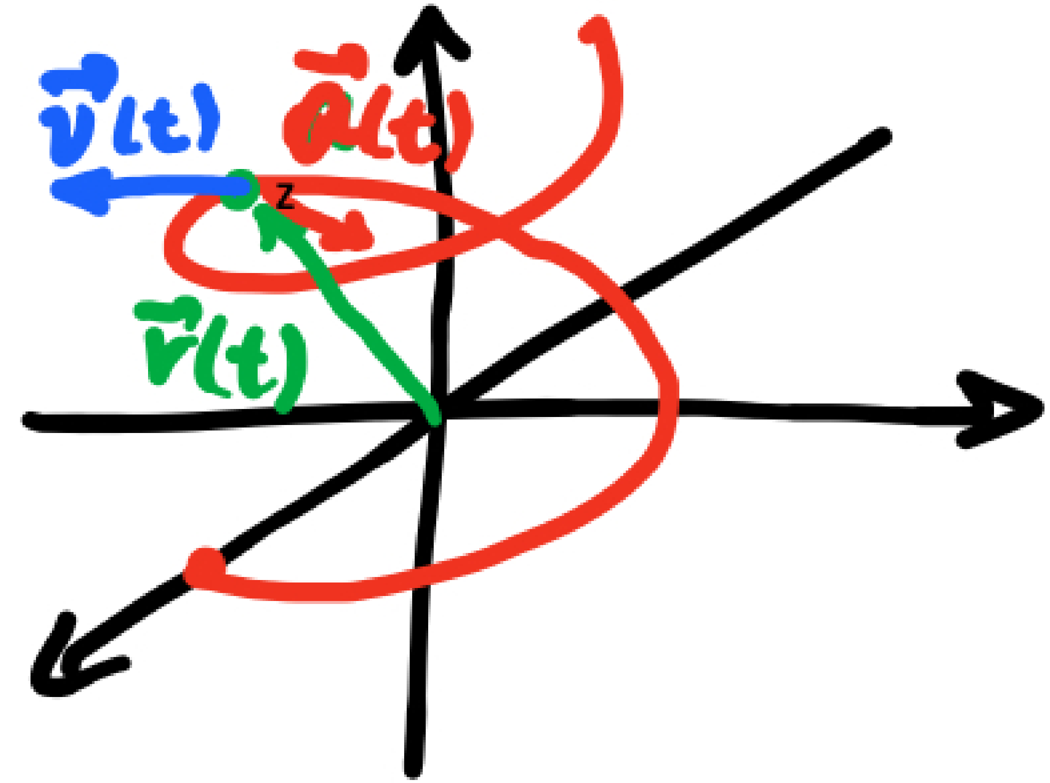
  $= \vec{r}\,'(t_0)$

- speed at $t = t_0$ is the magnitude of the velocity vector $= \|\vec{v}(t_0)\|$

- define acceleration of particle as derivative of velocity:

  $\vec{a}(t) = \vec{v}\,'(t) = \vec{r}\,''(t)$

Example Find velocity, speed and acceleration of $\vec{r}(t) = (\cos t, \sin t, t)$



$$\vec{v}(t) = (-\sin t, \cos t, 1)$$

$$\|\vec{v}(t)\| = \sqrt{(-\sin t)^2 + (\cos t)^2 + 1^2} = \sqrt{2}.$$

$$\vec{a}(t) = (-\cos t, -\sin t, 0).$$

Integral of a vector function

Define $$\int_a^b \vec{r}(t)\,dt = \int_a^b x(t)\,dt\,\hat{i} + \int_a^b y(t)\,dt\,\hat{j} + \int_a^b z(t)\,\hat{k}.$$

integrate componentwise

<u>Example</u> A particle starts at $\vec{r}(0)=(1,0,0)$ with velocity $\vec{v}(0)=\hat{\imath}+\hat{\jmath}+\hat{k}$ and acceleration $\vec{a}(t)=\cos t\,\hat{\imath}+2t\,\hat{\jmath}+\hat{k}$. Find velocity and position at time t.

$\vec{a}(t)=\vec{v}\,'(t) \Rightarrow \vec{v}(t)=\int \vec{a}(t)\,dt =$

$= \sin t\,\hat{\imath} + t^2\,\hat{\jmath} + t\,\hat{k} + \underline{C}$.

vector

We have $C=\vec{v}(0)$, so $\vec{v}(t)=(\sin t+1)\,\hat{\imath} + (t^2+1)\,\hat{\jmath} + (t+1)\,\hat{k}$.

$\vec{r}(t$

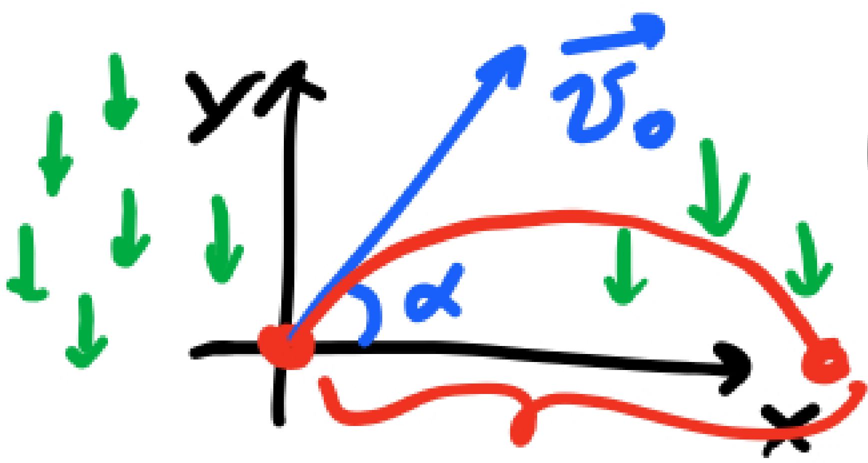
$\vec{v}(t)=\vec{r}\,'(t) \Rightarrow \vec{r}(t)=\int \vec{v}(t)\,dt =$

$(-\cos t+t)\,\hat{\imath} + (t^3/3+t)\,\hat{\jmath} + \left(\frac{t^2}{2}+t\right)\hat{k} + \underline{C'}$

$\cancel{C' = \vec{r}(0)}$, ~~so~~ $\vec{r}(0) = -\hat{\imath} + C' \Rightarrow C' = (2,0,0)$

$\vec{r}(t) = (-\cos t + t + 2)\,\hat{\imath} + \left(\frac{t^3}{3}+t\right)\hat{\jmath} + \left(\frac{t^2}{2}+t\right)\hat{k}$.

Application: projectile motion



projectile is fired with angle α and initial velocity $\vec{v}_0$.

assuming the only external force is gravity (no air resistance)

find position $\vec{r}(t)$.

Newton's second law of motion : $\vec{F}(t) = m\vec{a}(t)$

force $\vec{F}(t)$ acts on a particle of mass m producing acceleration $\vec{a}(t)$.

$$\vec{F} = m\vec{a} = -mg\,\hat{j} \Rightarrow \underline{\vec{a}(t) = -g}\,\hat{j}$$