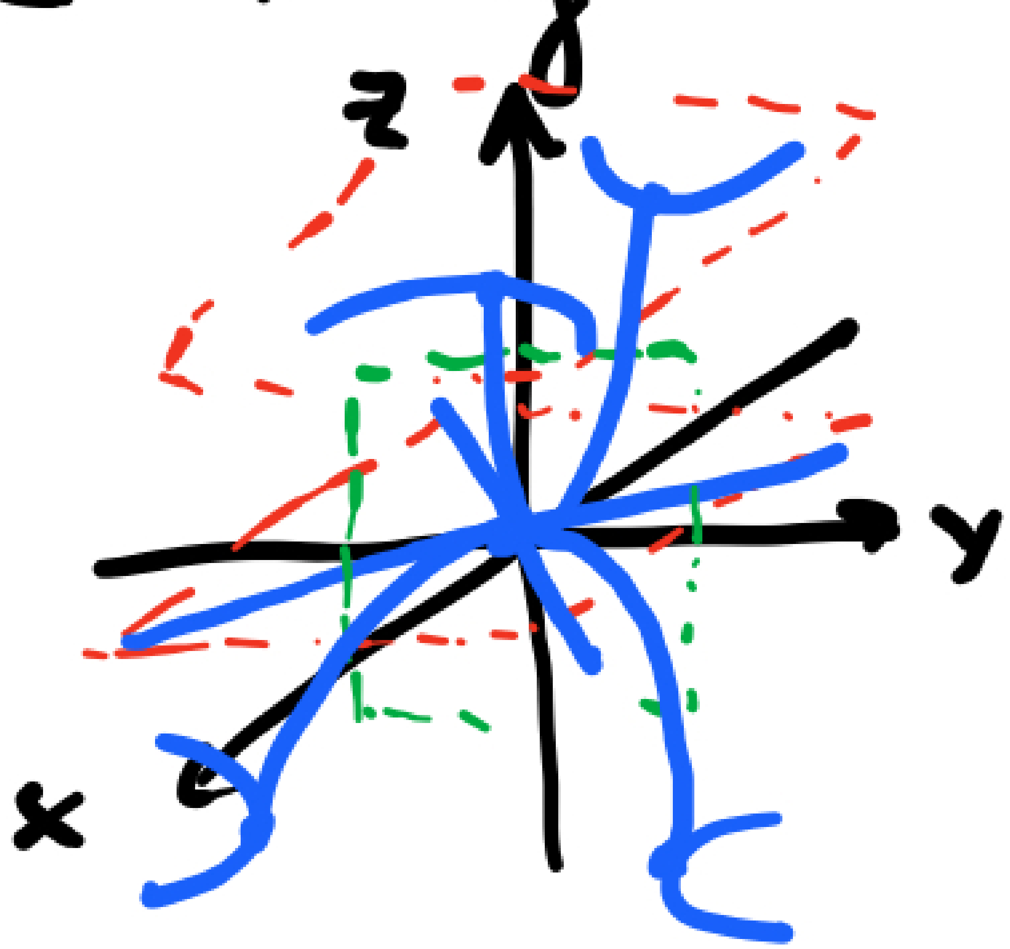
$z = x^2 - y^2$



$z = 0 \qquad 0 = x^2 - y^2$

$y^2 = x^2$

$\begin{cases} y = x \\ y = -x \end{cases}$

$z = 1 \qquad x^2 - y^2 = 1$

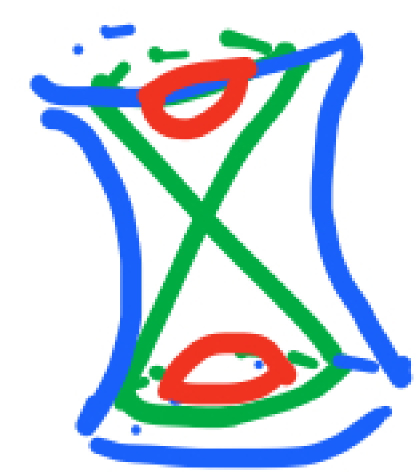
$z = -1 \qquad y^2 - x^2 = 1$

$x = 0 \qquad z = -y^2$

$y = 0 \qquad z = x^2$

"saddle"

hyperbolic paraboloid



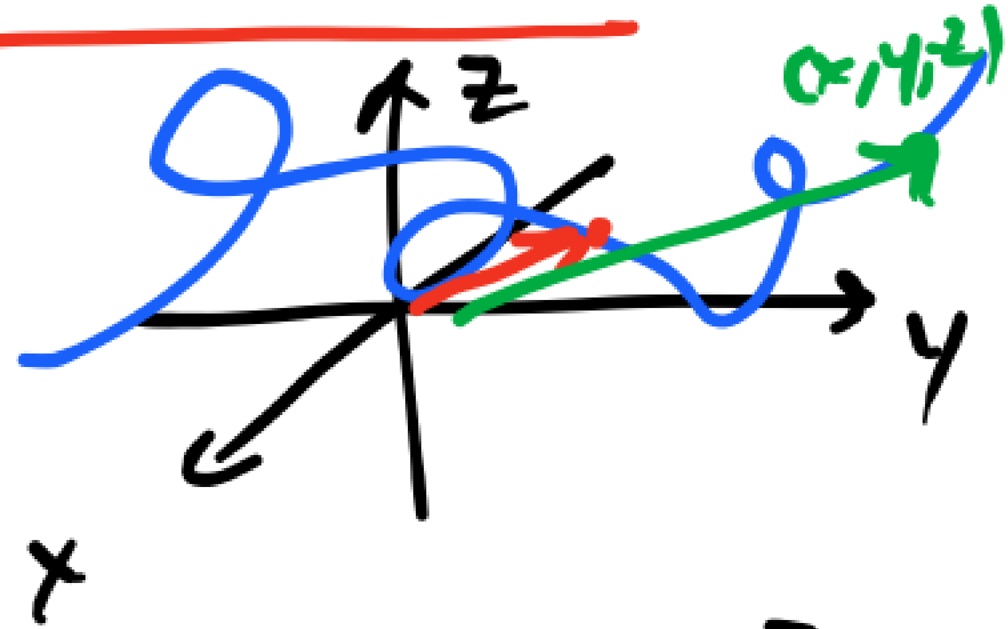
Example $x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$

$$(x-2)^2 - y^2 - (z+1)^2 + 3 \cancel{-4+1} = 0$$

double cone

13 Vector functions

Space Curves



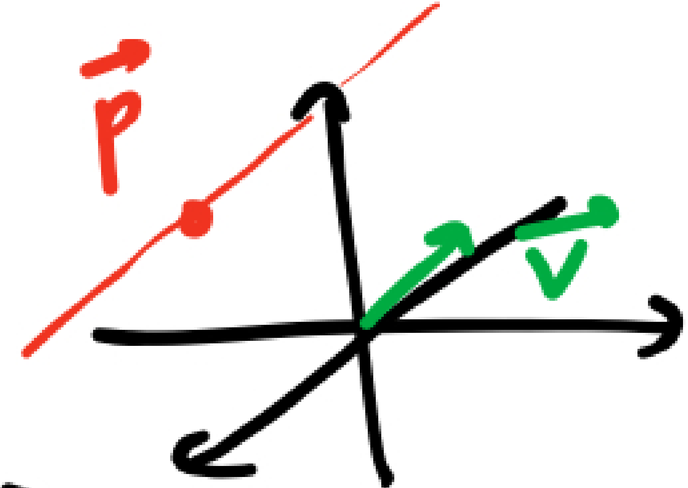
$$\begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$$

$$\vec{r}(t) = (x(t), y(t), z(t))$$

vector function

$$= x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$$

Example ① Line

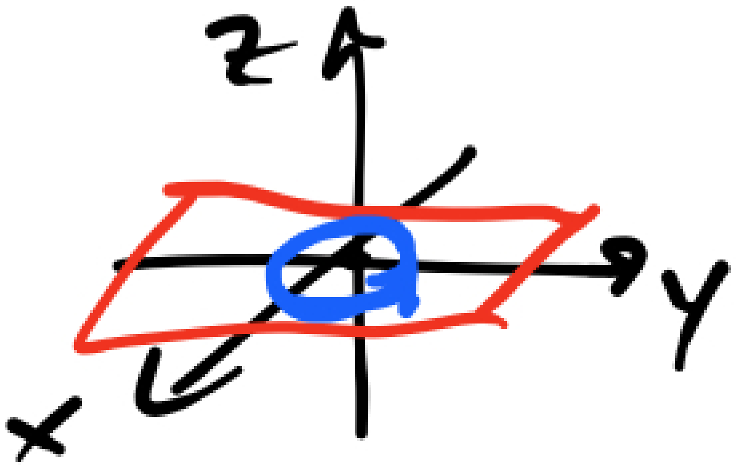


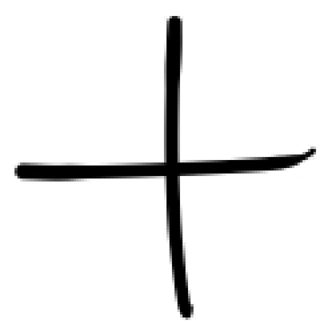
$$\vec{r}(t) = \vec{p} + t\vec{v} =$$

$$= (p_1, p_2, p_3) + t(v_1, v_2, v_3) =$$

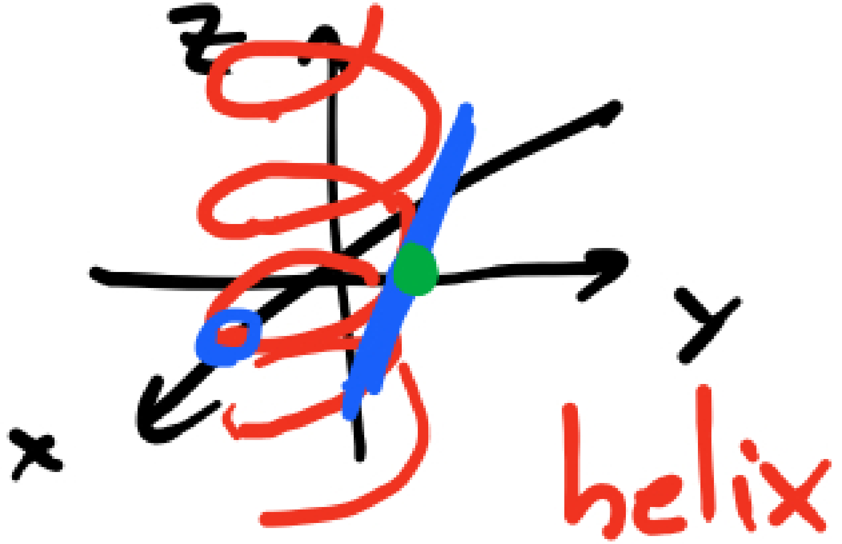
$$= (p_1 + tv_1, p_2 + tv_2, p_3 + tv_3).$$

② $\vec{r}(t) = (\cos t, \sin t, 0)$





(3) $\vec{r}(t) = (\cos t, \sin t, t)$



(4) $\vec{r}(t) = (\ldots, \ldots, \ldots)$ use software!

...

Calculus with space curves

* $\lim\limits_{t \to t_0} \vec{r}(t) = \lim\limits_{t \to t_0} (x(t), y(t), z(t)) =$

$$= \left(\lim_{t \to t_0} x(t), \lim_{t \to t_0} y(t), \lim_{t \to t_0} z(t) \right)$$

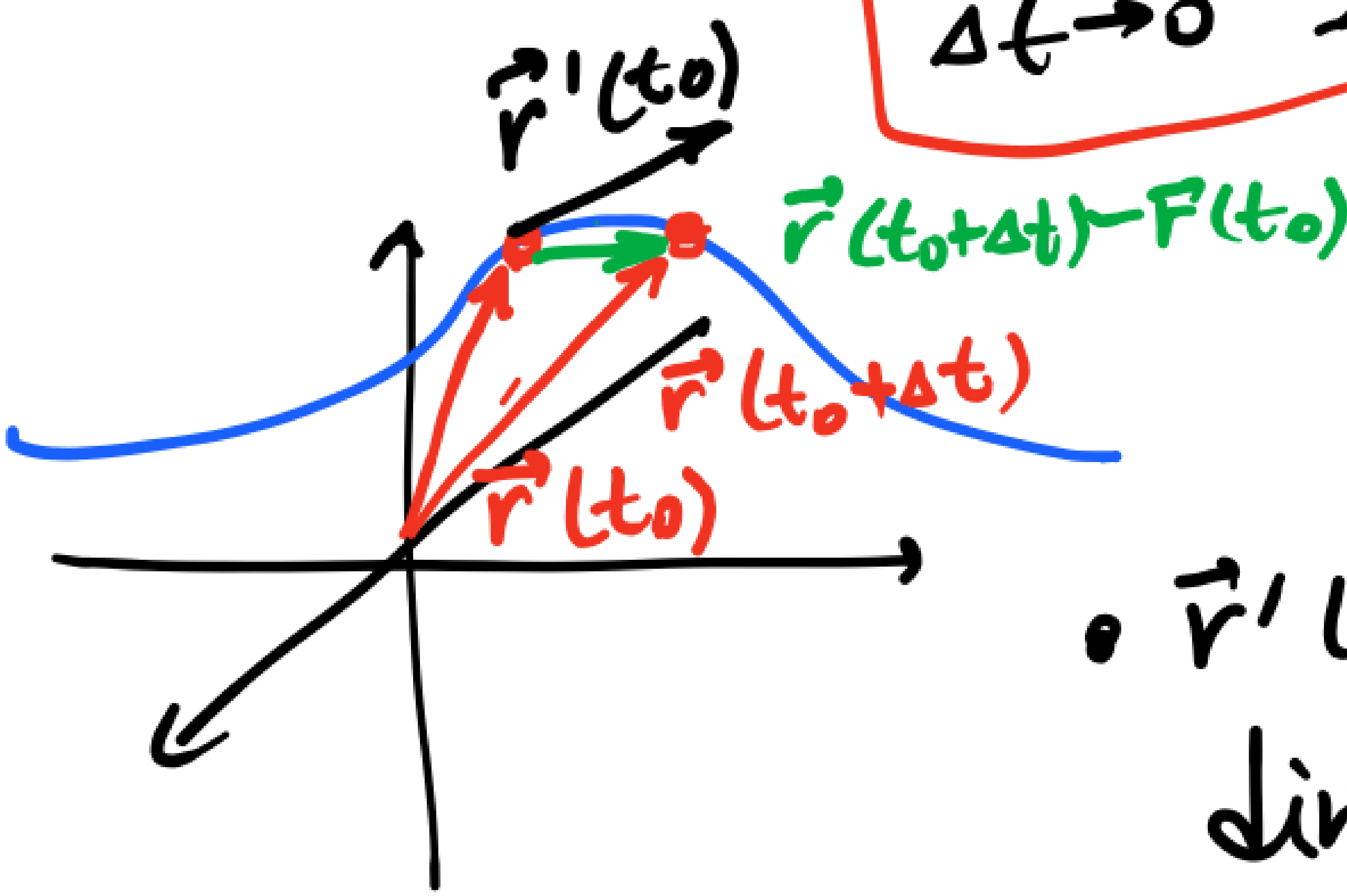
* we say $\vec{r}(t)$ is continuous at $t = t_0$ if $\lim\limits_{t \to t_0} \vec{r}(t) = \vec{r}(t_0)$

Derivative

$$\frac{\Delta \vec{r}}{\Delta t} \text{ as } \Delta t \to 0$$

Def^n The derivative of $\vec{r}(t)$ at $t=t_0$ is

$$\boxed{\lim_{\Delta t \to 0} \frac{\vec{r}(t_0+\Delta t)-\vec{r}(t_0)}{\Delta t} = \vec{r}\,'(t_0) = \left.\frac{d\vec{r}}{dt}\right|_{t=t_0}}$$



- $\vec{r}\,'(t_0)$ points in the direction of the tangent line of $\vec{r}(t)$ at $t=t_0$.

! If $\vec{r}(t) = (x(t), y(t), z(t))$, then $\vec{r}'(t) = (x'(t), y'(t), z'(t))$.

<u>Example</u> Find tangent line to helix at $t = \pi/2$.

$$\vec{r}'(t) = (-\sin t, \cos t, 1) \qquad \vec{r}_1(t) = (0, 1, \tfrac{\pi}{2})$$
$$\vec{r}'(\pi/2) = (-1, 0, 1) \qquad + t(-1, 0, 1)$$
$$= \underline{(-t, 1, \tfrac{\pi}{2} + t)}.$$

<u>Example</u> Exercise Is there a pt on twisted cubic $\vec{r}(t) = (t, t^2, t^3)$ where the tangent line is parallel to plane $z = 1 - 2x$?