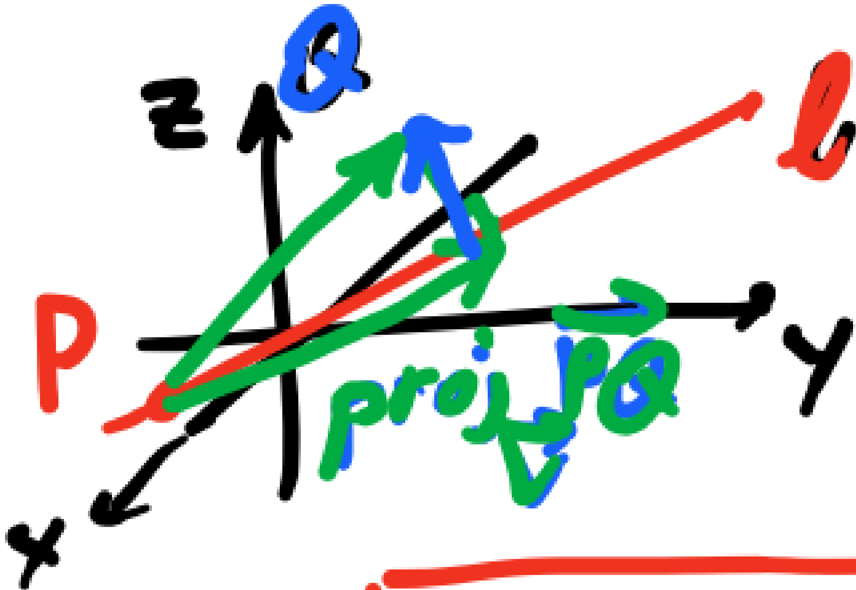
Lines & Planes

Problem Find distance from pt Q to a line ℓ

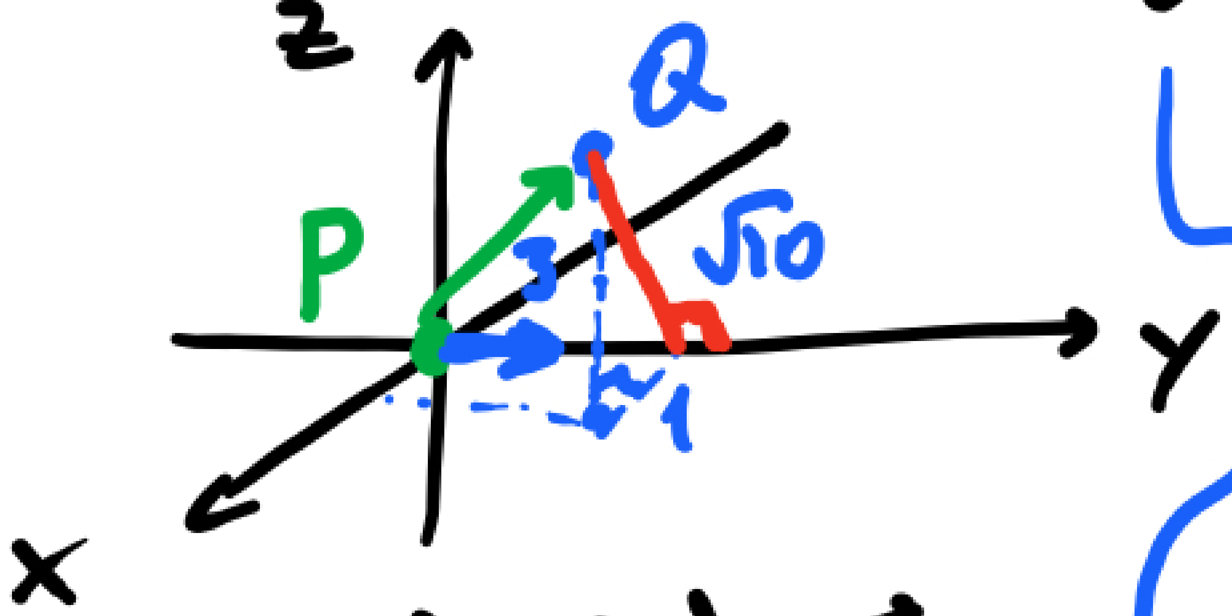


- take P on the line
- project $\vec{PQ}$ onto $\vec{v}$ - direction vector of ℓ.

Then $\boxed{\text{dist} = \|\vec{PQ} - \text{proj}_{\vec{v}}\vec{PQ}\|}$

Example $Q = (1,2,3)$

ℓ – y-axis



$$\vec{PQ} = \vec{Q} - \vec{P} =$$
$$= (1,2,3) - (0,0,0) =$$
$$= (1,2,3)$$

$\vec{v} = (0,1,0)$

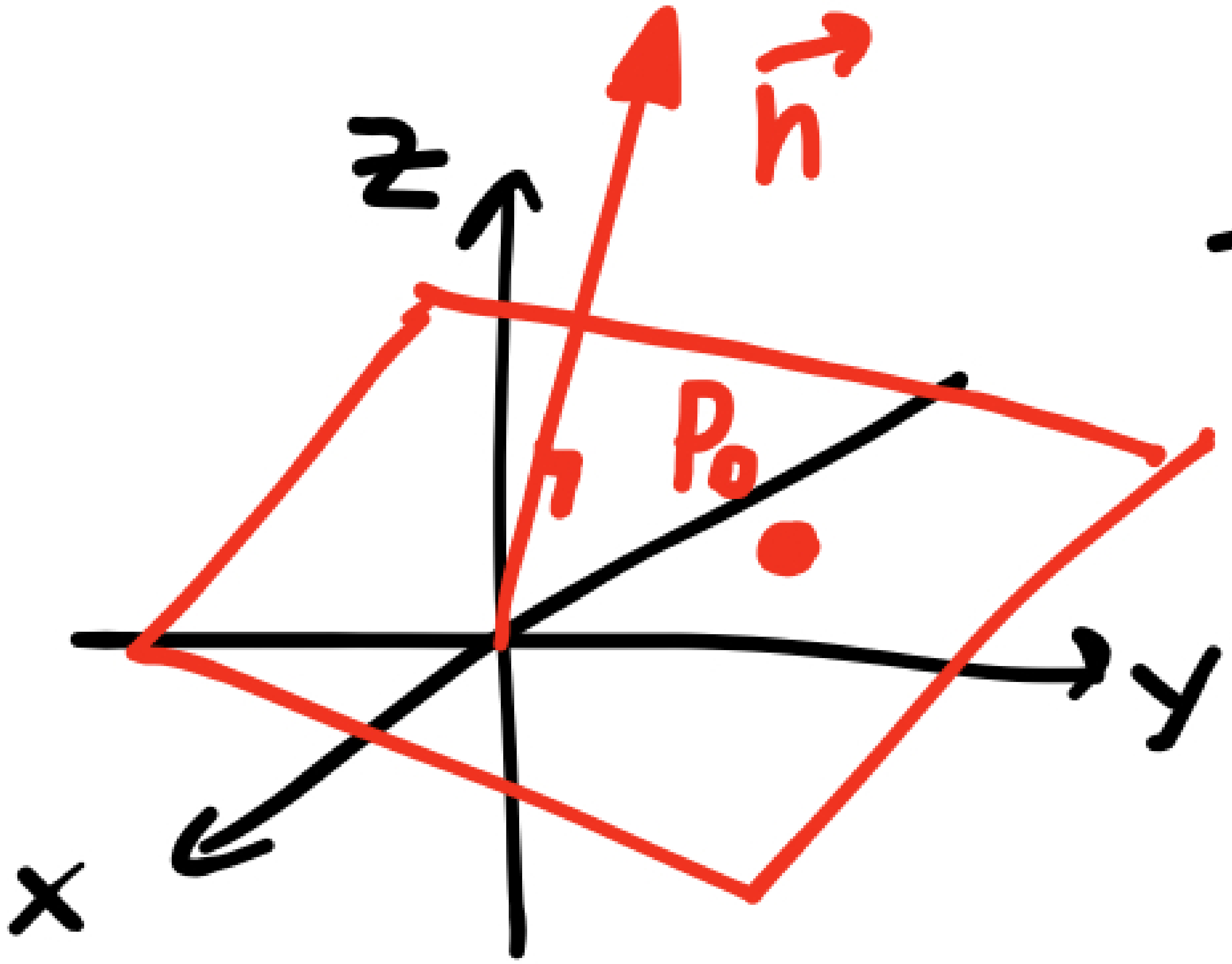
$\vec{P} = (0,0,0)$

$$d = \|\vec{PQ} - \operatorname{proj}_{\vec{v}} \vec{PQ}\| =$$

$$\operatorname{proj}_{\vec{v}} \vec{PQ} = \frac{\vec{PQ} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} =$$
$$= \frac{2}{1^2} \vec{v} = 2\vec{v} = (0,2,0)$$

$$d = \|(1,2,3) - (0,2,0)\| =$$
$$= \|(1,0,3)\| = \sqrt{10}.$$
$$'' \sqrt{1^2 + 0^2 + 3^2}$$

Planes in $\mathbb{R}^3$



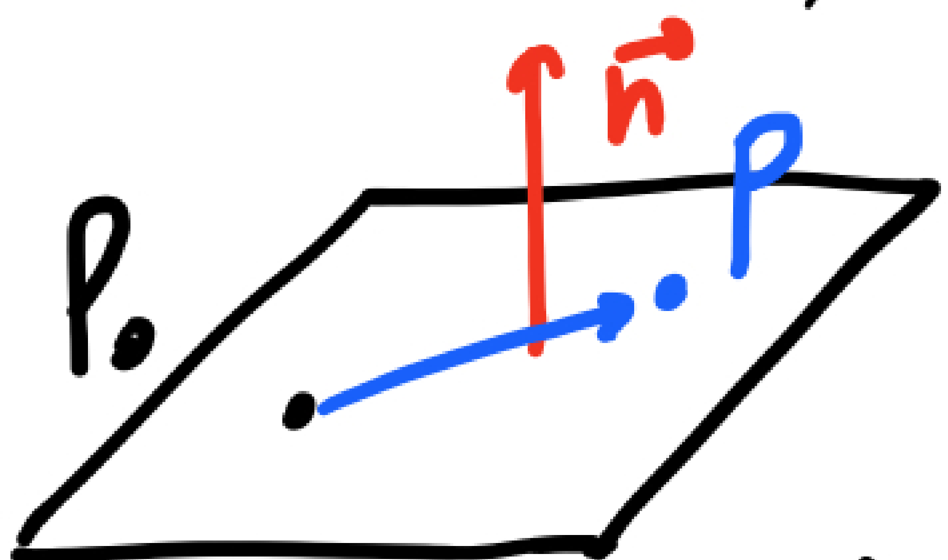
To specify a plane we need:

- a pt P_0 in it
- a normal vector $\vec{n}$

(vector perp to all vectors lying in the plane)

Equation $P_0 = (x_0, y_0, z_0)$

$\vec{n} = (a, b, c)$



$P = (x, y, z)$ - some other pt in the plane.

$$\vec{n} \perp \overrightarrow{P_0 P}$$

$$\iff \vec{n} \bullet \overrightarrow{P_0 P} = 0$$

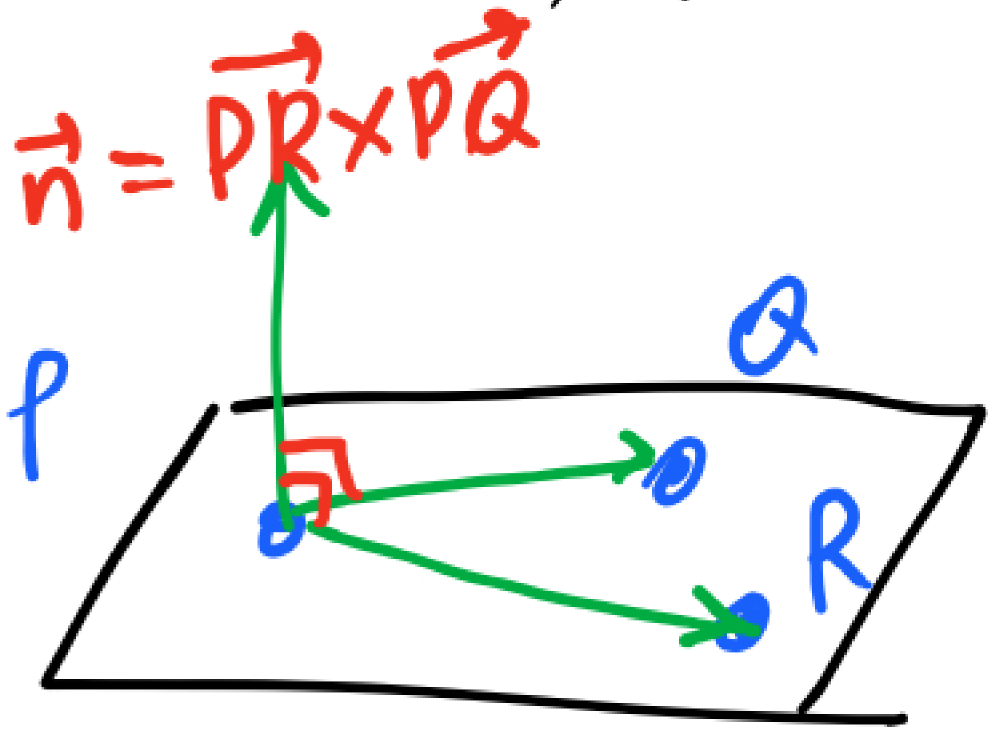
dot product

$$\iff \vec{n} \cdot (x - x_0, y - y_0, z - z_0) = 0$$

$$\iff \boxed{a(x - x_0) + b(y - y_0) + c(z - z_0) = 0}$$

Example Find eq^n of plane contains

$P=(3,2,-1)$, $Q=(1,-1,3)$,
$R=(3,-2,4)$



• $\vec{n} = \vec{PR} \times \vec{PQ} = (-1,-10,-8)$.
check

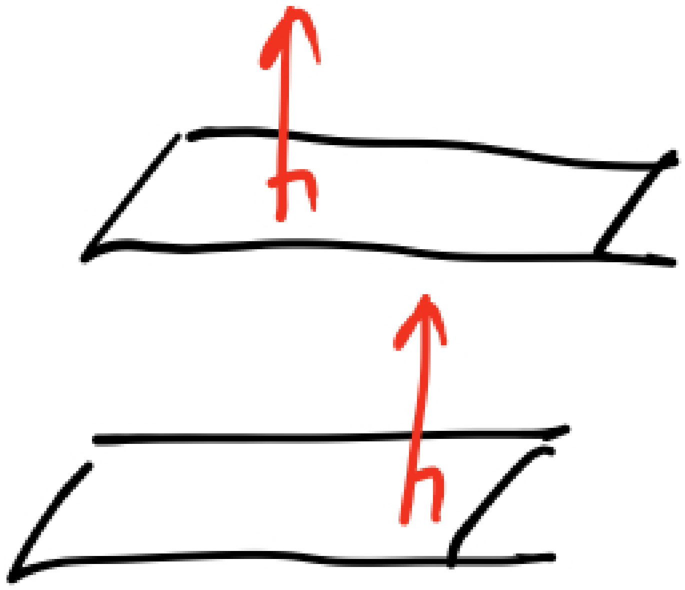
$\Rightarrow \; -(x-3) - 10(y-2) - 8(z+1) = 0$ check for Q, R.

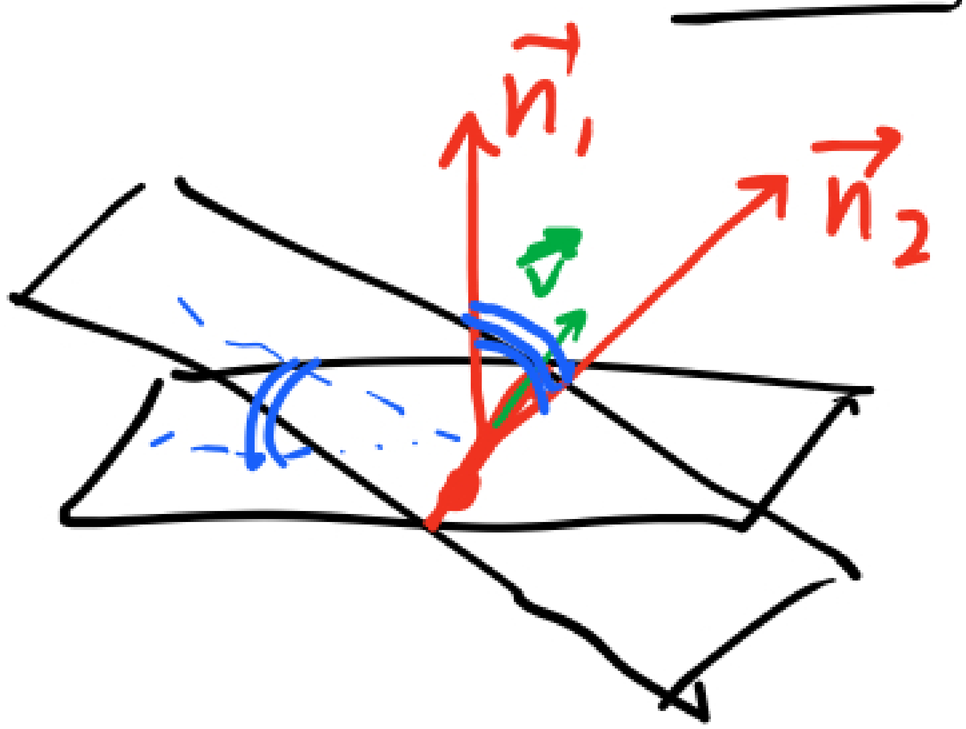
$\Leftrightarrow \; \boxed{x + 10y + 8z - 15 = 0}$

Remark: eq^n of plane in $\mathbb{R}^3$ can be written as $\boxed{ax+by+cz+d=0}$

Compare: eq^n of line in $\mathbb{R}^2$ can be written as $\boxed{ax+by+c=0}$

Example In $\mathbb{R}^3$ planes are either parallel or intersect in a line





$$\vec{n}_1 = c\,\vec{n}_2$$

$$\vec{v} = \vec{n}_1 \times \vec{n}_2$$

direction vector of line of intersection

Example

$$(*)\quad \begin{cases} 4x+4y-2z=9 \\ 2x+y+z=-3 \end{cases}$$

① find the line of intersection

② find the angle between the planes.

① $\vec{n}_1 = (4,4,-2)$

$\vec{n}_2 = (2,1,1)$

$\vec{v} = \vec{n}_1 \times \vec{n}_2 =$

$= (6,-8,-4)$.

Exercise: find a point P on the line of intersection.

② Hint: dot product.