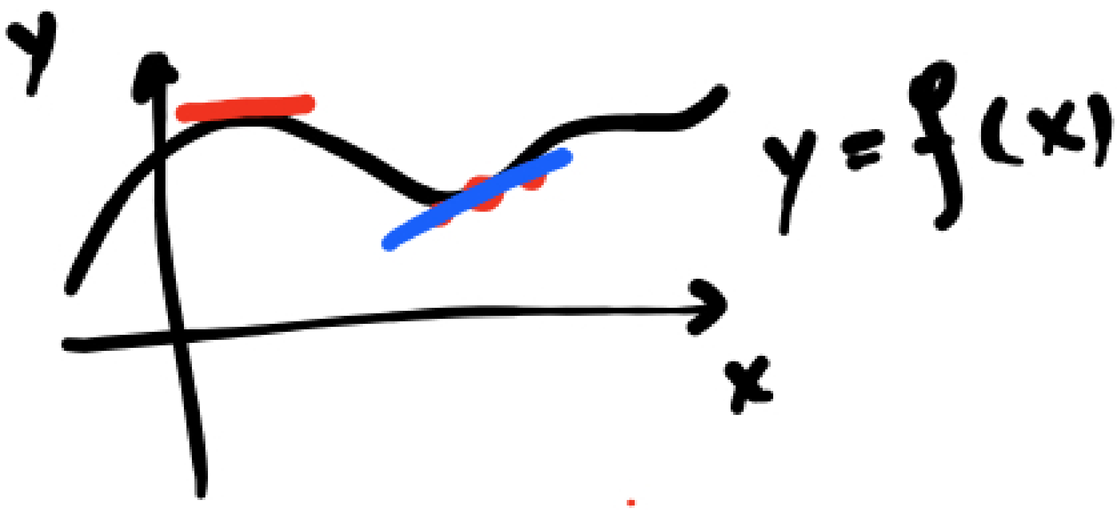
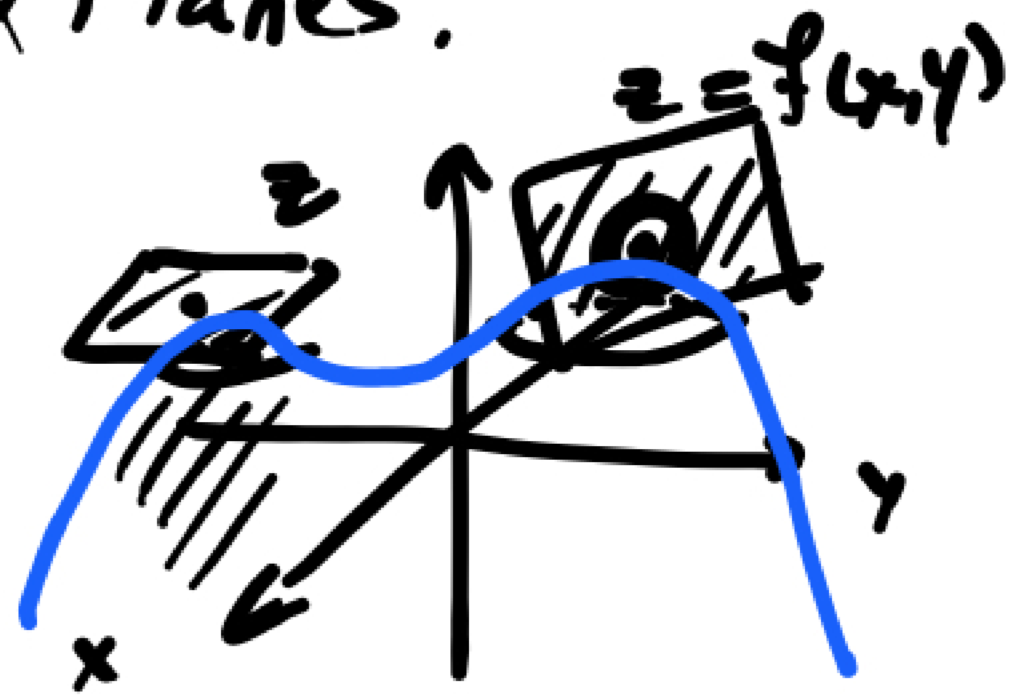
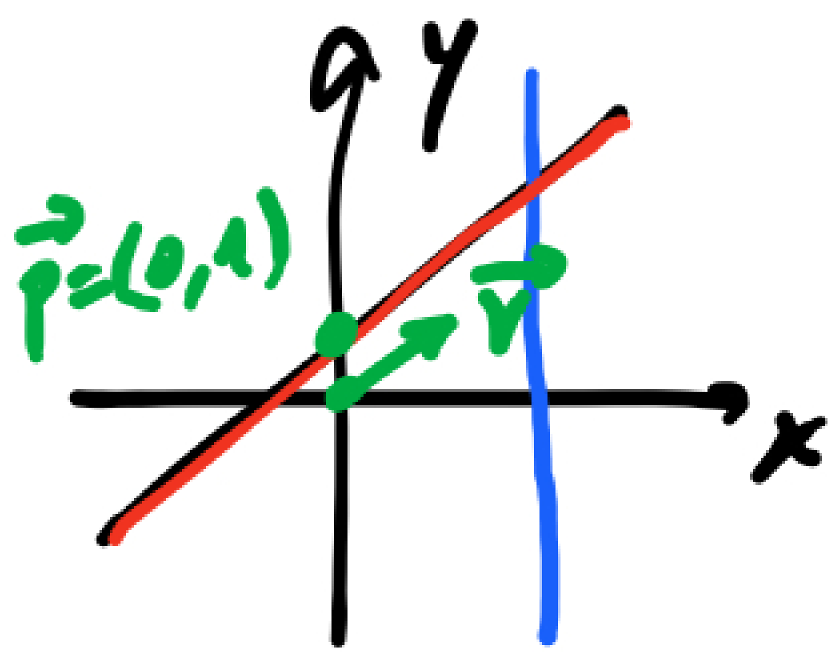
12.5. Lines & Planes.





Lines:

in $\mathbb{R}^2$:



$y = x + 1$

$x = 5$

$$\boxed{ax + by + c = 0}$$

Let $\vec{v}$ be a direction vector of the line $y = x + 1$

slope of $\vec{v}$ = slope of $y = x + 1$

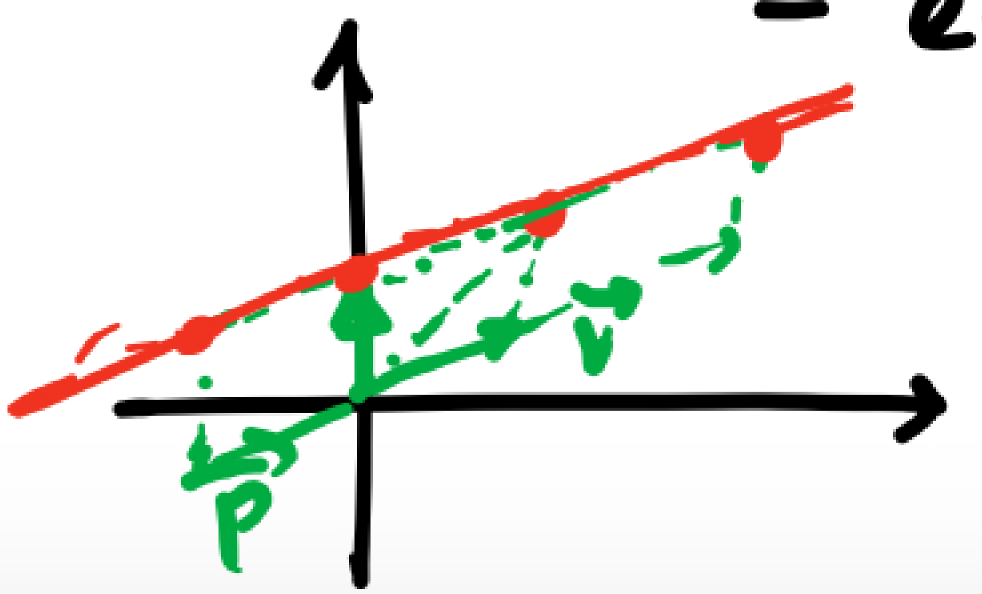
$\Rightarrow \vec{v} = (1,1)$.
e.g.

To determine the line we need:

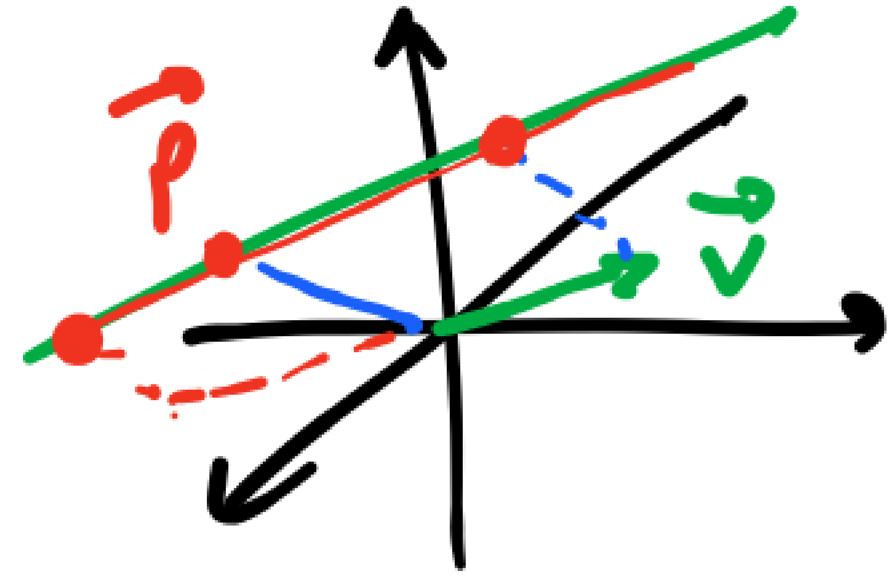
- a pt $\vec{p}$ on the line
- a direction vector $\vec{v}$

$$\boxed{\vec{r}(t) = \vec{p} + t\vec{v}}$$

- equation of a line in vector form



Works in $\mathbb{R}^3$!



Examplee $\quad y = 2x + 1$

$$\vec{p} = (0, 1)$$
$$\vec{v} = (1, 2)$$

$$\vec{r}(t) = \vec{p} + t\vec{v} = (0,1) + t(1,2) =$$
$$= (0,1) + (t, 2t) = (\underbrace{t}_{x=}, \underbrace{2t+1}_{y=}).$$

$y = 2t+1 = 2x+1.$

* different choice of $\vec{p}, \vec{v}$:

$\vec{p} = (1,3)$

$\vec{v} = (2,4)$

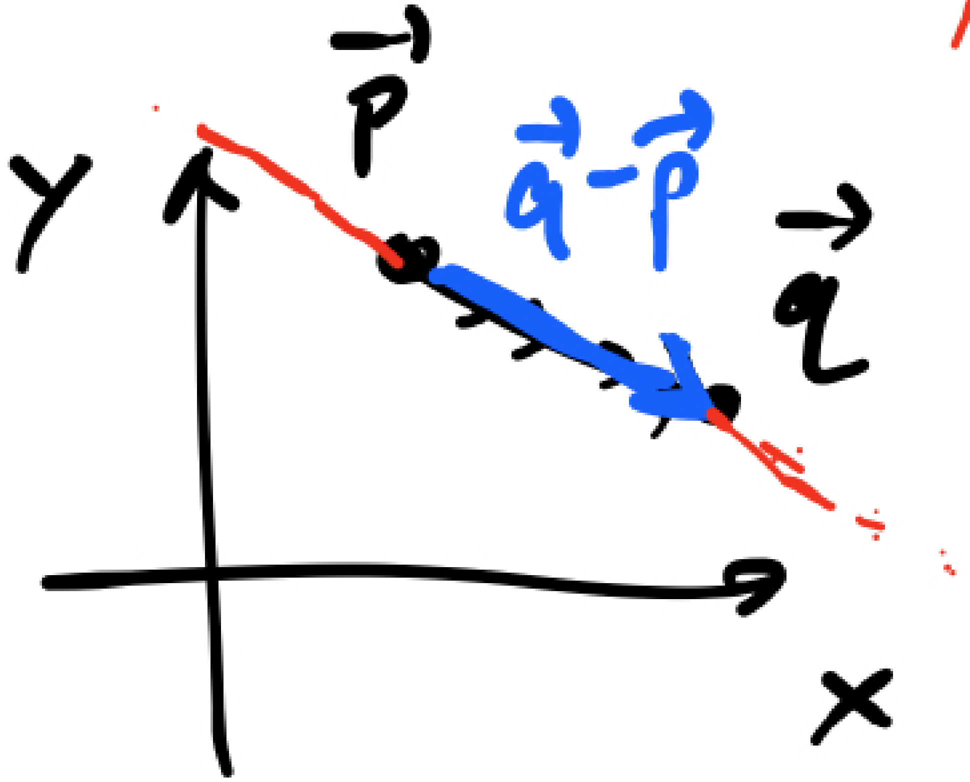
$\vec{r}(t) = \vec{p} + t\vec{v} = (1,3) + t(2,4) =$

$(\underbrace{2t+1}_{x=}, \overbrace{4t+3}^{y_i})$.

different parametrization

$y = 4t+3 = 2(2t+1)+1 = 2x+1.$

$\vec{\ }$

★ 

Recall $\quad 0 \le t \le 1$

Line segment: $\vec{r}(t) = (1-t)\vec{p} + t\vec{q}$

$$\vec{r}(t) = (1-t)\vec{p} + t\vec{q} = \vec{p} - t\vec{p} + t\vec{q} = \underline{\vec{p}} + t(\underline{\vec{q}-\vec{p}}).$$

★ Find ~~eqn~~ vector form of a line in $\mathbb{R}^3$ passing through

$(1,0,1)$ and $(2,4,-1)$

"$\vec{p}$" "$\vec{q}$"

$$\vec{p} = (1,0,1)$$

$$\vec{v} = \vec{q} - \vec{p} = (1, 4, -2)$$

$$\vec{r}(t) = (1,0,1) + t\,(1,4,-2) =$$

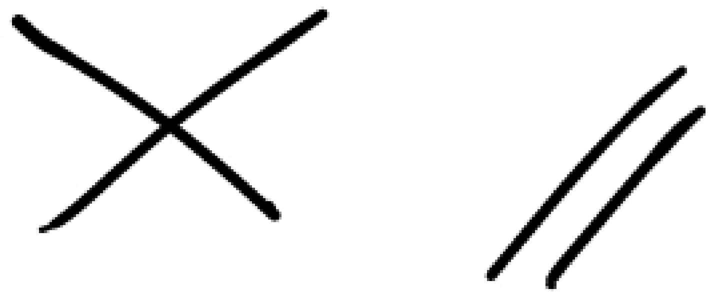
$$= (\underbrace{1+\underline{t}}_{x=}, \underbrace{4\underline{t}}_{y=}, \underbrace{1-\underline{2}t}_{z=}).$$

Another way: "Symmetric eq's"

solve for $\underline{t}$: $t = x-1$, $t = \frac{y}{4}$, $t = \frac{1-z}{2}$

$$\boxed{x-1 = \frac{y}{4} = \frac{1-z}{2}}$$

In $\mathbb{R}^2$



In $\mathbb{R}^3$

intersect parallel

skew

Example Determine whether lines are parallel, skew or intersect.

$$\begin{cases} x = 1 + 2t \\ y = 1 + 3t \\ z = -t \end{cases} \qquad \begin{cases} x = 2t \\ y = 2 + 3t \\ z = 4 - t \end{cases}$$

- parallel

Admit the same direction vector $(2,3,-1)$

$$\begin{cases} x = 3 + t \\ y = 2 + t \\ z = -1 + t \end{cases} \qquad \begin{cases} x = 6 + t \\ y = 5 + t \\ z = 2 + t \end{cases}$$

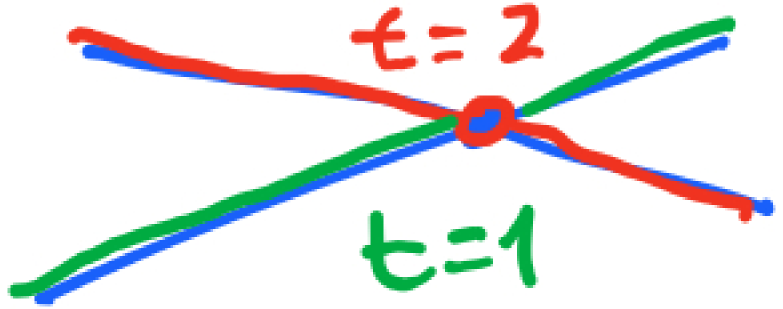
- same line!

$(1,2,3)$

$$\begin{cases} x = 1+t \\ y = 2t \\ z = 1+3t \end{cases}$$

$(3,2,$

$$\begin{cases} x = 3t \\ y = 2t \\ z = 2+t \end{cases}$$



$$\begin{cases} 1+t = 3s \Rightarrow t = s = 1/2 \\ 2t = 2s \Rightarrow t = s \\ 1+3t = 2+s \end{cases}$$

$\Rightarrow$ intersect at $(3/2, 1, 5/2)$.

$$\begin{cases} x = 1+t \\ y = 2t \\ z = 1+3t \end{cases}$$

$$\begin{cases} x = 3t \\ y = 2t \\ z = 1+t \end{cases}$$

$\to$ skew

Q