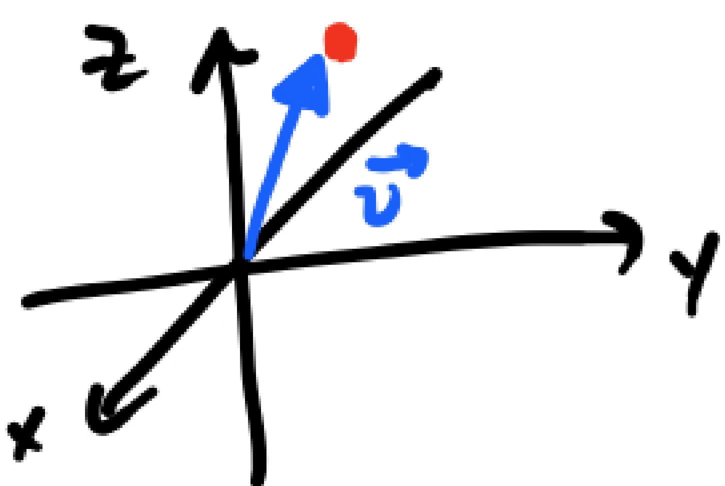
Last time:

- Vectors in $\mathbb{R}^2$ (and $\mathbb{R}^3$)
  - addition
  - scalar multiplication

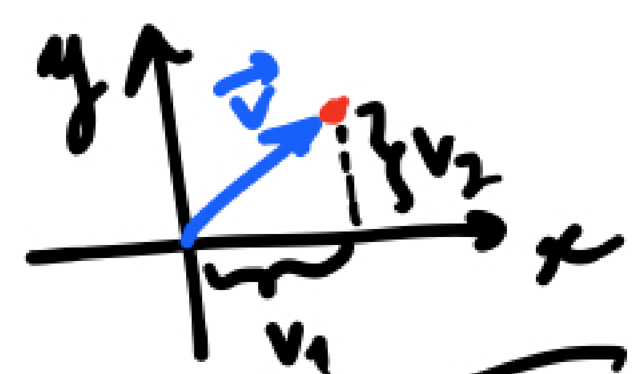
Vector norm (or length)

norm of $\vec{v}$

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$



In $\mathbb{R}^2$:

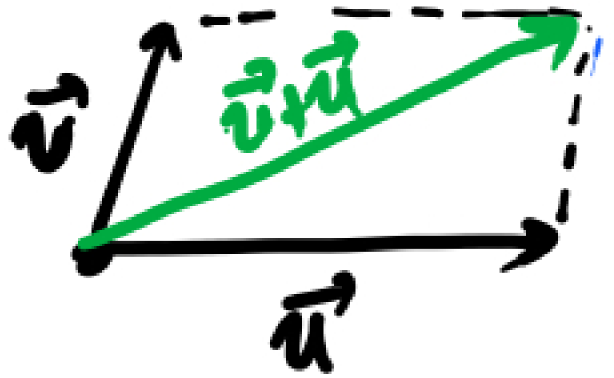


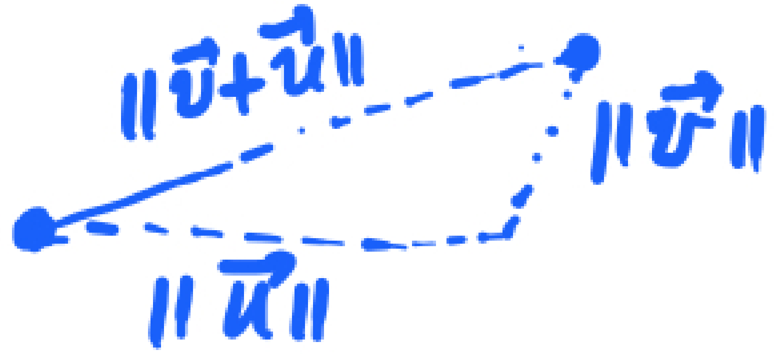
$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2}.$$

properties of norm: ⓪ $\|\vec{v}\| \geq 0$.

① $\|\vec{v}+\vec{u}\| \leq \|\vec{v}\| + \|\vec{u}\|$

(triangle inequality)

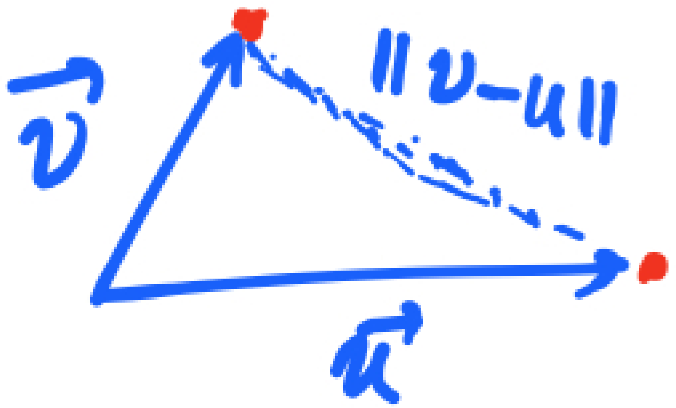




② $\|c\vec{v}\| = |c|\,\|\vec{v}\|$

$\|2\vec{v}\| = 2\|\vec{v}\|$ $\|-v\| = \|v\|$.

③ $\|\vec{v}-\vec{u}\|$ = distance between $\vec{v}$ and $\vec{u}$



Example

Example What is the eqⁿ of a sphere in $\mathbb{R}^3$ centered at $(3,0,-2)$ of radius 4?

(x,y,z) is on sphere iff $\|(x,y,z)-(3,0,-2)\|=4$.

$$\|(x-3,\ y,\ z+2)\|=4$$

$$\sqrt{(x-3)^2+y^2+(z+2)^2}=4$$

$$\boxed{(x-3)^2+y^2+(z+2)^2=16}$$

Can we multiply vectors?

$$\vec{v} \cdot \vec{u} = \cancel{(v_1, v_2, v_3) \cdot (u_1, u_2, u_3) = (v_1 u_1, v_2 u_2, v_3 u_3)}$$

Dot product of vectors

$$\vec{v} \cdot \vec{u} = \underbrace{v_1 u_1 + v_2 u_2 + v_3 u_3}_{\text{number}}$$

observations: ① $\vec{v} \cdot \vec{v} = v_1^2 + v_2^2 + v_3^2 = \|\vec{v}\|^2$

(2) $\vec{v} \bullet \vec{u} = \|\vec{v}\| \cdot \|\vec{u}\| \cos\theta$,

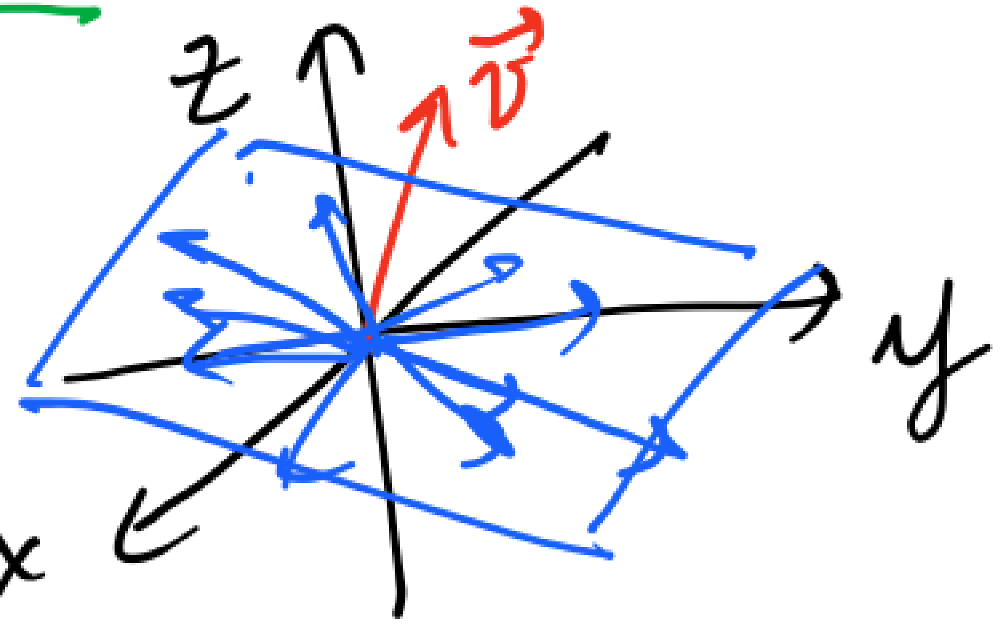
where θ is the angle between $\vec{v}$ and $\vec{u}$ $(0 \leq \theta \leq \pi)$

(2') $\vec{v} \cdot \vec{u} = 0$

$\Leftrightarrow \vec{v} \perp \vec{u}$

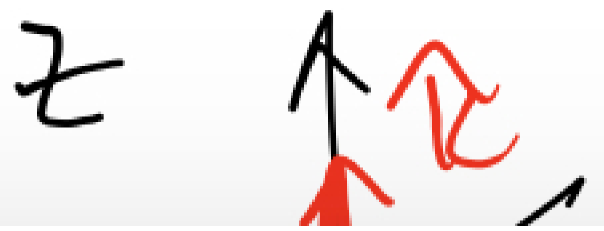
Example Find a vector $\vec{u}$ that is $\perp$ to $\vec{v} = (1,2,3)$.

eg. $\vec{u} = (1,1,-1)$ $\quad \vec{v} \cdot \vec{u} = (1,2,3) \cdot (1,1,-1) = 1 + 2 + (-3) = 0.$



All pts (x,y,z) s.t. $(x,y,z) \cdot (1,2,3) = 0$

form a plane $\boxed{x + 2y + 3z = 0}$



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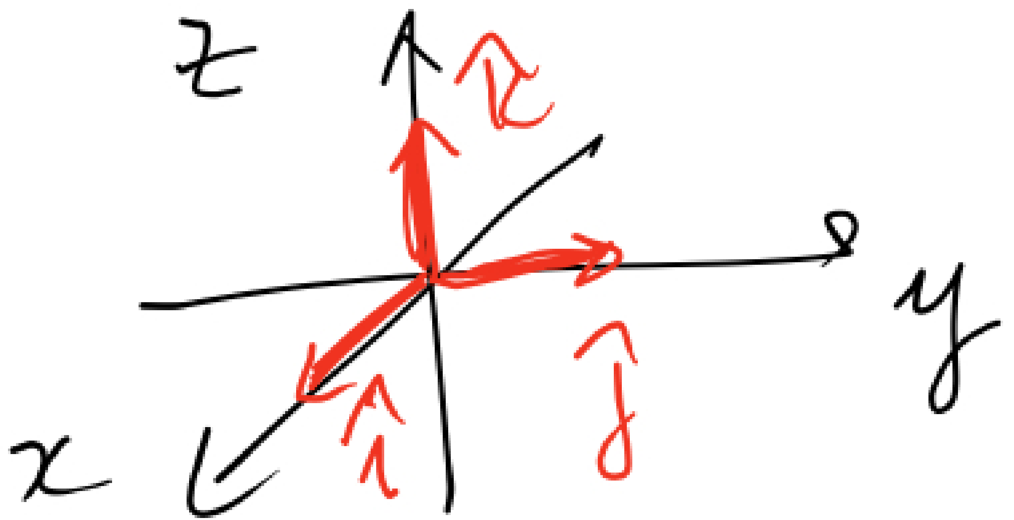

$$\left.\begin{aligned} \hat{i} &= (1,0,0) \\ \hat{j} &= (0,1,0) \\ \hat{k} &= (0,0,1) \end{aligned}\right\}$$ "standard basis vectors for $\mathbb{R}^3$"