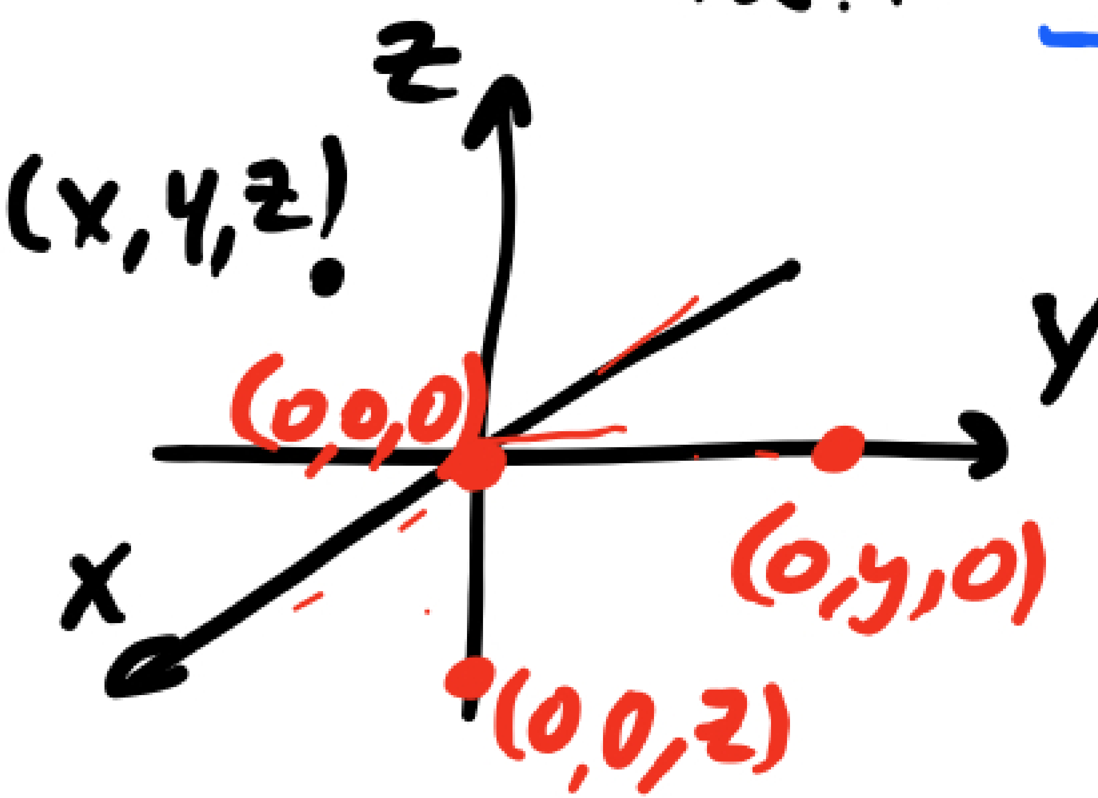
12. Vectors & Geometry (in $\mathbb{R}^2$ and $\mathbb{R}^3$)

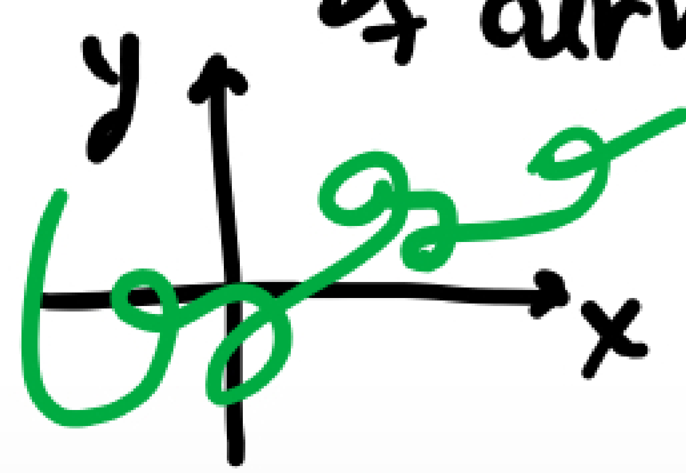
12.1. 3D Space ($\mathbb{R}^3$)

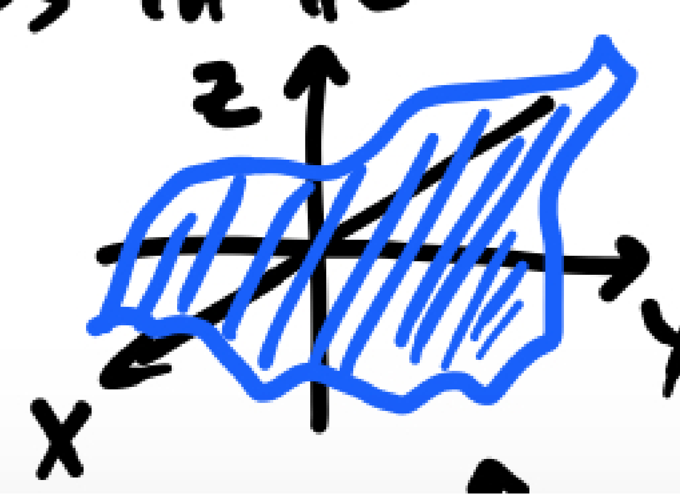
$$\mathbb{R}^3 = \{(x,y,z) \mid x,y,z \in \mathbb{R}\}$$

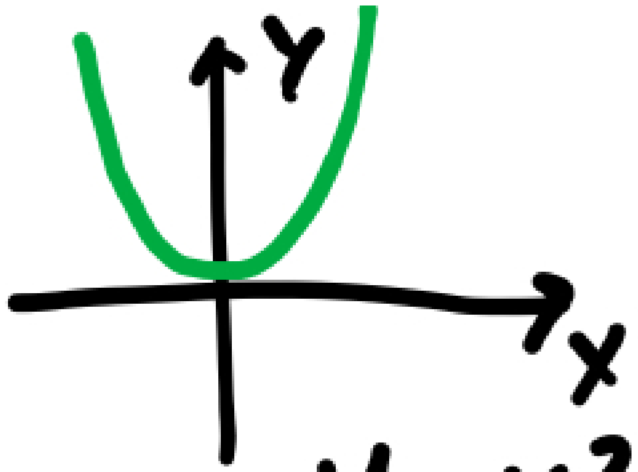


Surfaces in $\mathbb{R}^3$:

Idea: genaralize notion of curves in $\mathbb{R}^2$

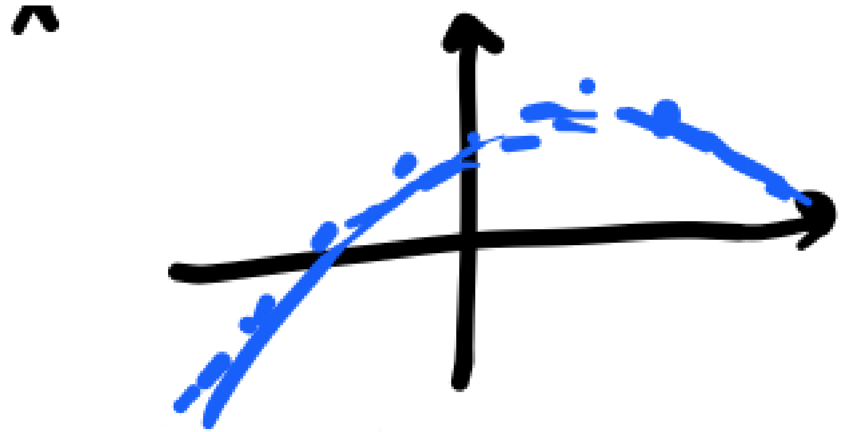






$y = x^2$

$y - x^2 = 0$



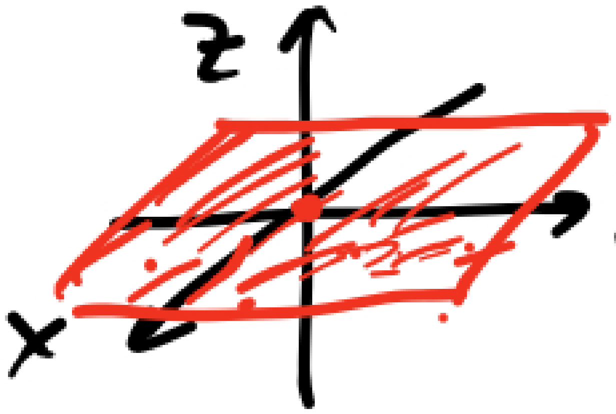
(x, y) such that $F(x,y) = 0$.

(x, y, z) such that $F(x,y,z) = 0$

$z - x^2 - y^2 = 0$

Examples

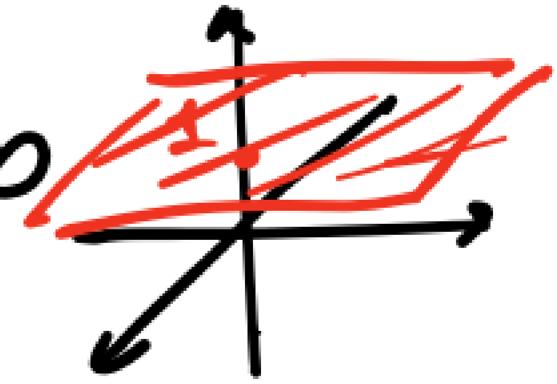
$z = 0$ (pts (x,y,z) st. $z=0$)



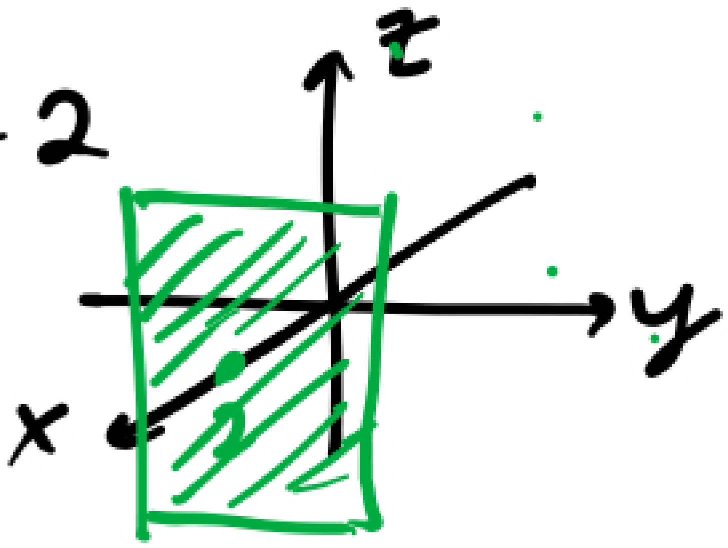
xy-plane

$z = 1$

$z - 1 = 0$



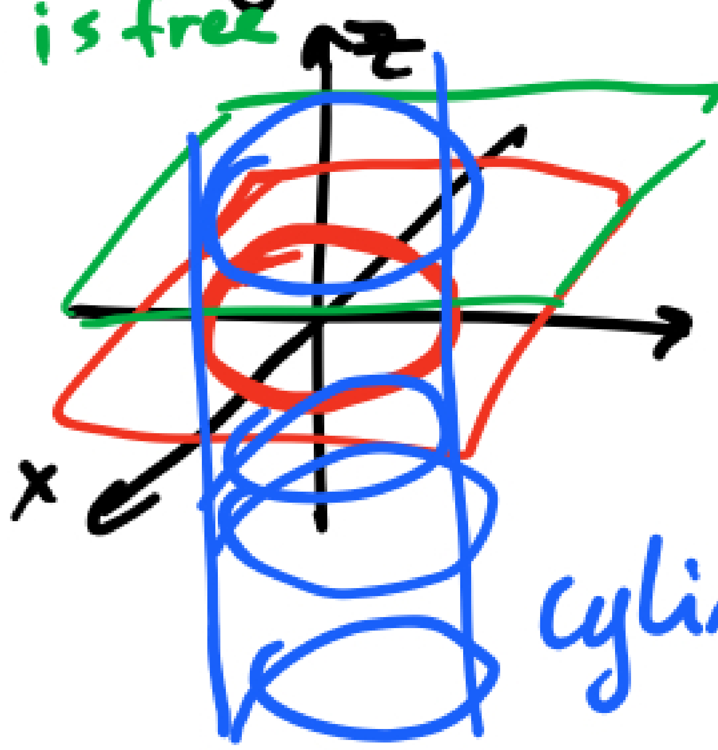
$x = 2$



$x^2+y^2=1$

$x^2+y^2-1=0$

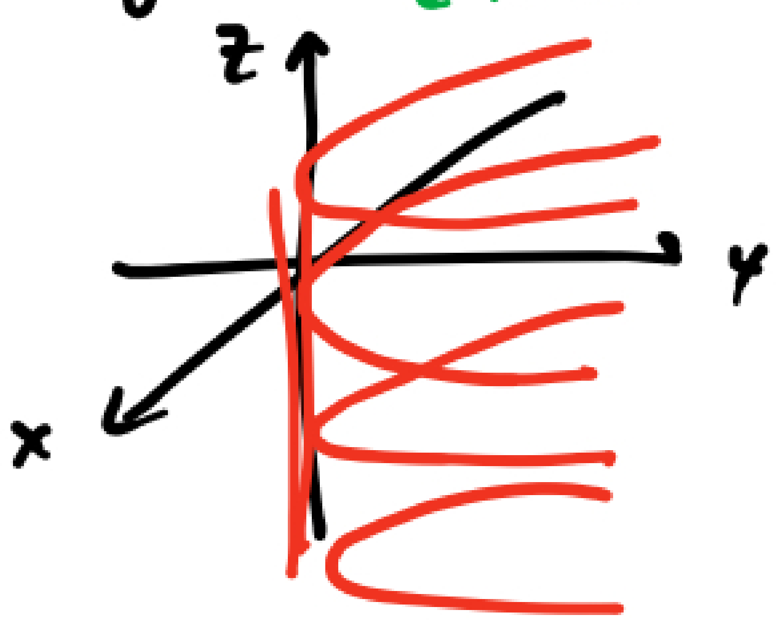
z is free

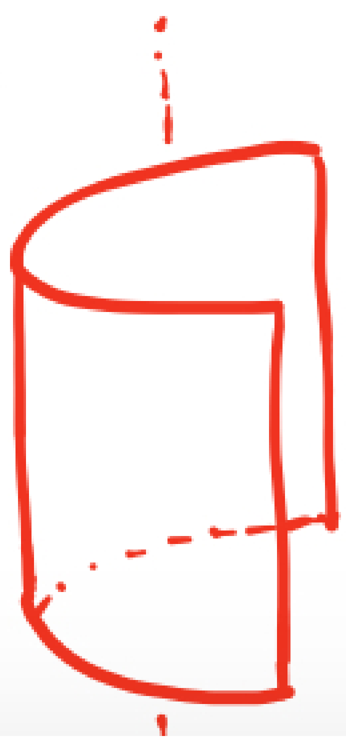


cylinder

$y = x^2$

z is free

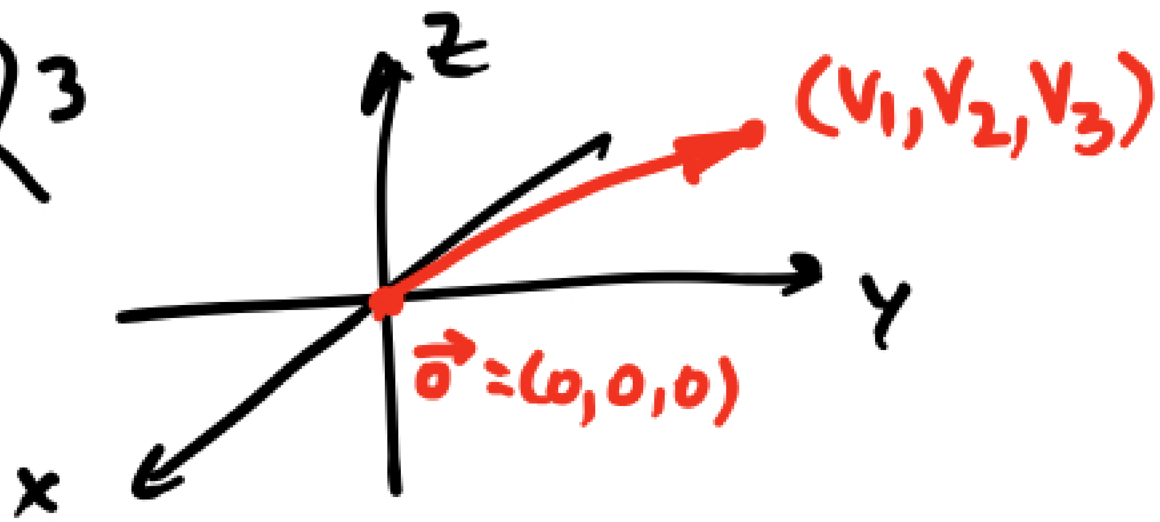




parabolic cylinder

12.2+ Vectors (in $\mathbb{R}^2$ and $\mathbb{R}^3$)

$\mathbb{R}^3$



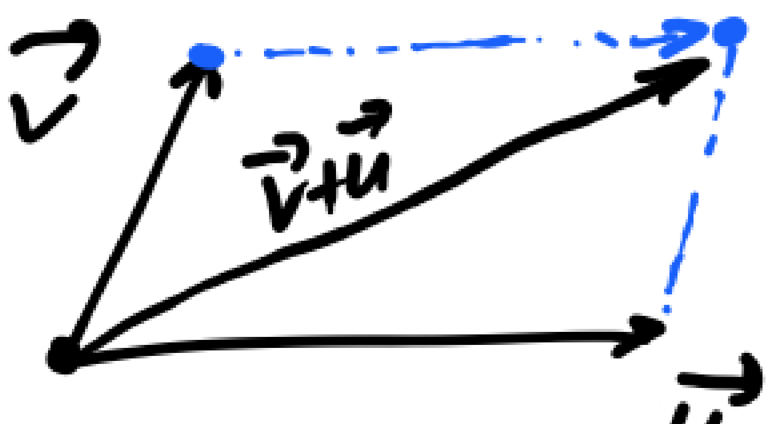
A vector is a point $\vec{v} = (v_1, v_2, v_3)$

(same idea works for $\mathbb{R}^n$)

operations:

(1) Addition: $\vec{v} + \vec{u} = (v_1, v_2, v_3) + (u_1, u_2, u_3) =$

$= (v_1 + u_1, v_2 + u_2, v_3 + u_3)$



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② Scalar multiplication

if $\vec{v} \in \mathbb{R}^3$ and $c \in \mathbb{R}$

$$c \cdot \vec{v} = c(v_1, v_2, v_3) =$$
$$= (cv_1, cv_2, cv_3)$$

$\vec{v}$

$c\vec{v}$ (if $c > 1$)

$c\vec{v}$ (if $0 < c < 1$)

$c\vec{v}$ (if $c < 0$)

We can define

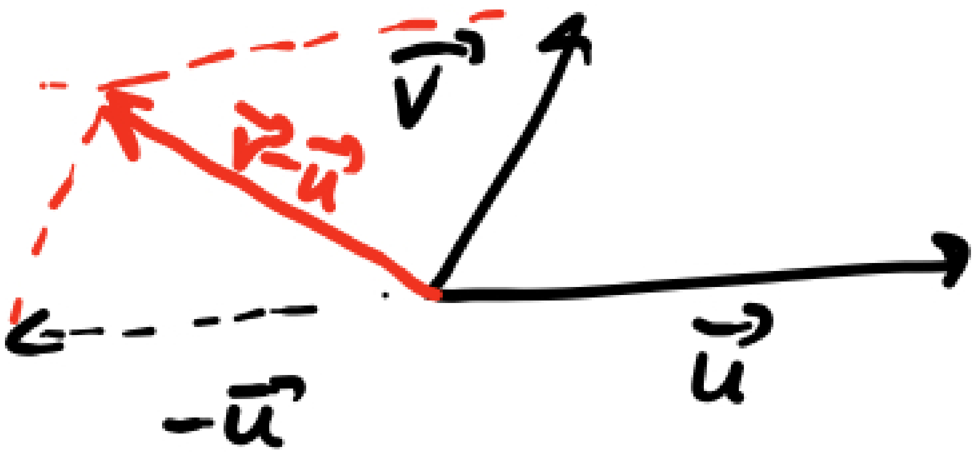
$$-\vec{v} = (-v_1, -v_2, -v_3)$$

$\vec{v}$

$-\vec{v}$

We can define subtraction of vectors:

$$\vec{v} - \vec{u} = \vec{v} + (-\vec{u}) = (v_1 - u_1, v_2 - u_2, v_3 - u_3)$$



Properties:

(1) $\vec{v}+\vec{u}=\vec{u}+\vec{v}$

(2) $c(\vec{v}+\vec{u})=c\vec{v}+c\vec{u}$

(3) – (8)