Mathematical Introduction to Game Theory

Assignment 6, due November 13

Problem 1 of 5. Prove that in a two-person general sum game, the expected payoff of any player at any Strategic Equilibrium (mixed or pure) can not be smaller than the safety level of this player.
Hint: A player can always switch to his/her optimal strategy if this would not be the case.

Problem 2 of 5. Find all the Nash equilibria in the game with the matrix

$$\begin{pmatrix} (0,4) & (3,0) \\ (2,2) & (1,3) \end{pmatrix}.$$

Problem 3 of 5. Let (A, B) be a *constant-sum game*, i.e. there exists a constant L such that for every i, j, $a_{ij} + b_{ij} = L$. Prove that the payoff of R is the same at any Nash equilibrium. Prove that the same is true for C.
Hint: If $L = 0$, it is a zero-sum game, and we can use Minimax Theorem. For other L, just subtract it from all elements of one of the matrices.

Problem 4 of 5. Consider the 3×3 two-person non-zero-sum game with payoff matrix:

$$\begin{pmatrix} (3,2) & (3,0) & (2,2) \\ (1,0) & (2,3) & (0,3) \\ (0,2) & (0,0) & (3,2) \end{pmatrix}$$

Find all the Nash equilibria (pure and mixed) of the game.

Problem 5 of 5. Consider the following model of duopoly.

The market has capacity A for a certain good. The production cost of each unit equal to C_1 for the company I and $C_2 < C_1$ for the company II. The price of the product is equal to $A - Q$, where Q is the total number of the units produced. Company I makes a decision about the number of units it will produce and informs Company II about its decision. The company II then makes the decision about the number of units it will produce.

Analyze the model by finding all the Nash equilibria, and comparing the consumer prices and profits with the case of the monopoly of the company I.