Mathematical Introduction to Game Theory

Assignment 5, due October 23

Problem 1 of 5. Let the matrix of an $n \times n$ game A satisfy the following conditions:

$$\text{For each } i,\ 1 \le i \le n: \quad \sum_{j=1}^{n} A_{ij} = M$$

and

$$\text{For each } j,\ 1 \le j \le n: \quad \sum_{i=1}^{n} A_{ij} = M$$

Find the value of the game and show that the strategy

$$\begin{pmatrix} 1/n \\ 1/n \\ \vdots \\ 1/n \end{pmatrix}$$

is optimal for both players.

Problem 2 of 5. Solve the following zero-sum game, i.e. find the value of the game and all optimal strategies for both players

$$\begin{pmatrix} 12 & -4 & 0 & 0 \\ 0 & 12 & 0 & 0 \\ 0 & 0 & 18 & -9 \\ 0 & 0 & 0 & 18 \end{pmatrix}.$$

Justify your answer.

Problem 3 of 5. Find the value of Blotto-Kije game with two posts to capture, where Blotto has five units, and Kije only has three units.

Problem 4 of 5. Consider the following game. Each of the two players tosses a coin. The coin of the first player is fair, but the coin of the second player is not, it lands "Heads" with probability 1/4 and "Tails" with probability 3/4. The outcomes of their tosses are seen by players but not shown to the opposing player. The first player can put \$1 on the table or fold (and thus end the game without any payoffs). The second player then can either call (and put \$1 on the table) or fold (and also end the game with no payoff). If neither of the players folds the first player takes the money on the table if the both tossed coins gave the same value. Otherwise, this amount is taken by the second player.

Draw the Kuhn tree, convert to the matrix form, find the value of the game and some optimal strategies for both players.

Problem 5 of 5. Consider the following game: R chooses 0 or 1 and announces her choice. C chooses 0 or 1, but stays quiet. R then again chooses 0 or 1. If the sum of the three numbers is even, R gets \$1 from C, otherwise C gets \$1 from R.

Draw the Kuhn tree, convert to the matrix form, find the value of the game and some optimal strategies for both players.

Hint: Problem 1 (with $M = 0$) might help you here.