## Mathematical Introduction to Game Theory

Assignment 8, due November 29

**Problem 1 of 5.** Let  $(N, v_1)$  and  $(N, v_2)$  be two games in coalitional form with non-empty cores. Prove that for any positive  $\lambda > 0$  and  $\mu > 0$ , the function  $v := \lambda v_1 + \mu v_2$  is a characteristic function and that the game (N, v) has a non-empty core.

**Problem 2 of 5.** Find the characteristic function of the 3-person game in strategic form when the payoff vectors are:

If I chooses 1:	If I chooses 2:
$ \begin{pmatrix} (2,7,-2) & (3,0,1) \\ (-1,6,3) & (3,-2,1) \end{pmatrix} $	$\begin{pmatrix} (-1,2,4) & (1,3,3) \\ (7,5,-4) & (3,-2,1) \end{pmatrix}$

**Problem 3 of 5.** (Oil Market game.) Country 1 has oil which it can use to run its transport system at a profit of a per barrel. Country 2 wants to buy the oil to use in its manufacturing industry, where it gives a profit of b per barrel, while Country 3 wants it for food manufacturing where the profit is c per barrel. Let  $a < b \le c$ .

- (1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.
- (2) Describe all the imputations.
- (3) Compute the core of the game.
- (4) Find the Shapley Value.

**Problem 4 of 5.** A toy costs \$40 and consists of three parts: I, II, III. There is one manufacturer of part I, two manufacturers of part II, and three manufacturers of part III. No part of the toy can be sold separately.

- (1) Describe the problem as a game in coalitional form, i.e. define the characteristic function.
- (2) Describe all the imputations.
- (3) Compute the core of the game.
- (4) Find the Shapley Value.

**Problem 5 of 5.** Prove that the game in coalitional form (N, v) is inessential if and only if -v is a characteristic function.