Mathematical Introduction to Game Theory

Assignment 3, due October 4

Problem 1 of 5. Consider a zero-sum game with the matrix

$$\begin{pmatrix} 0 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 \\ 1 & -2 & 0 & -1 \\ -2 & 0 & 0 & 1 \end{pmatrix}.$$

Find Chris' optimal response to the $\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, 0\right)$ strategy of Ruth.

Problem 2 of 5. Solve the following zero-sum game, i.e. find the value of the game and all optimal strategies for both players

$$\begin{pmatrix} -1 & 5 \\ 8 & 2 \end{pmatrix}.$$

Problem 3 of 5. Consider a zero-sum game with the matrix

$$\begin{pmatrix} 0 & 1 & 1 & a \\ -1 & 0 & 2 & 0 \\ 1 & -2 & 0 & -1 \\ -2 & 0 & 0 & -2 \end{pmatrix}.$$

where a is a parameter. For which values of a does the game have saddle point?

Problem 4 of 5. Solve the following zero-sum game, i.e. find the value of the game and all *pure* optimal strategies for both players

$$\begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & 5 & 8 \\ 0 & 1 & 0 & 3 \end{pmatrix}.$$

Problem 5 of 5. Let $\mathbf{z} \in \mathbb{R}^n$, $\mathbf{z} \neq \mathbf{0}$, and $c \in \mathbb{R}$. Denote by $H(\mathbf{z}, c)$ the closed subspace $\left\{\mathbf{x} \in \mathbb{R}^n \ : \ \mathbf{z}^T\mathbf{x} \leq c\right\}$.

Let $K \subset \mathbb{R}^n$ be a closed convex set.

Denote

$$\tilde{K} = \cap_{(K \subset H(\mathbf{z},c))} H(\mathbf{z}, c),$$

i.e. $\tilde{K}$ is the intersection of all closed subspaces containing K.

(1) Prove that if $\mathbf{x} \in K$, then $\mathbf{x} \in \tilde{K}$ and thus $K \subset \tilde{K}$.

(2) Using the Separation Theorem, prove that if $\mathbf{x} \notin K$ then $\mathbf{x} \notin \tilde{K}$, and thus $\tilde{K} \subset K$. Conclude that $K = \tilde{K}$.