MAT406H5F. Assignment 5, due November 9

Problem 1 of 5

For the following game, find the safety levels of both players, all Pareto optimal strategies, and all pure strategic equilibria

$$\begin{pmatrix} (0,2) & (1,-1) & (2,1) \\ (2,2) & (-1,2) & (4,1) \\ (-1,3) & (2,-2) & (0,2) \\ (1,1) & (2,2) & (2,0) \end{pmatrix}.$$

Problem 2 of 5

Contestants I and II start the game with \$200 and \$400 dollars respectively. Each must decide to pass or gamble, not knowing the choice of the other. A player who passes keeps the money he/she started with. If Player I gambles, he wins \$400 with probability 1/2 or loses \$200 with probability 1/2. If Player II gambles, she wins or loses \$400 with probability 1/2 each. These outcomes are independent. Then the contestant with the higher amount at the end wins a bonus of \$200. If both amounts are equal, nobody gets a bonus.

1. Draw the Kuhn tree.
2. Put into strategic form.
3. Find the safety levels.
4. Find all the strategic equilibria.

Problem 3 of 5

Prove that in a two-person general sum game, the expected payoff of any player at any Strategic Equilibrium (mixed or pure) can not be smaller than the safety level of this player. **Hint:** A player can always switch to his/her optimal strategy if this would not be the case.

Problem 4 of 5

Find all the Nash equilibria in the game with the matrix

$$\begin{pmatrix} (0,4) & (3,0) \\ (2,2) & (1,3) \end{pmatrix}.$$

Problem 5 of 5

Let (A, B) be a *constant-sum game*, i.e. there exists a constant L such that for every i, j, $a_{ij} + b_{ij} = L$. Prove that for every two Nash equilibria the payoffs of R are the same.
Hint: If $L = 0$, it is a zero-sum game, and we can use Minimax Theorem. For other L, just subtract it from all elements of one of the matrices.