(1) A staircase of n steps contains coins on some of the steps. Let $(x_1, x_2, \ldots, x_n)$ denote the position with x_j coins on step j, $j = 1, \ldots, n$. A move of *Staircase Nim* consists of moving any positive number of coins from any step, j, to the next lower step, $j-1$. Coins reaching the ground (step 0) are removed from play. A move taking, say, x chips from step j, where $1 \leq x \leq x_j$, and putting them on step $j-1$, leaves $x_j - x$ coins on step j and results in $x_{j-1} + x$ coins on step $j-1$. The game ends when all coins are on the ground. Players alternate moves and the last to move wins.

Show that the SG-function of a position $(x_1, x_2, \ldots, x_n)$ in the Staircase Nim is equal to the Nim-sum of the numbers of coins at the odd positions:

20 points $x_1 \oplus x_3 \oplus \ldots x_{2k_1}$, where $k = \frac{n+1}{2}$ if n is odd and $k = \frac{n}{2}$ if n is even.

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(2) Player I draws a card at random from a full deck of 52 cards. After looking at the card, he bets either 1 or 3 that the card he drew is a face card (king, queen or jack, probability 3/13). Then Player II either concedes or doubles. If she concedes, she pays I the amount bet (no matter what the card was). If she doubles, the card is shown to her, and Player I wins twice his bet if the card is a face card, and loses twice his bet otherwise.

(a) Draw the Kuhn tree.

(b) Find the equivalent strategic form.

20 points (c) Solve the game.

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(3) Find the safety levels, maxmin strategies, and all Nash Equilibria for the game given in the matrix form by the following bi-matrix.

$$\begin{pmatrix} (1,4) & (2,7) & (3,2) & (2,6) \\ (5,2) & (0,3) & (4,3) & (6,1) \\ (2,3) & (3,4) & (5,2) & (3,5) \end{pmatrix}$$

20 points

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(4) Consider a two-person cooperative game given by the following matrix

$$\begin{pmatrix} (2,0) & (3,-3) & (2,-1) & (10,-9) & (0,0) \\ (7,5) & (3,1) & (3,2) & (2,1) & (-1,2) \\ (2,3) & (0,0) & (1,1) & (4,5) & (-1,4) \\ (-1,0) & (8,7) & (5,6) & (3,2) & (-1,5) \end{pmatrix}.$$

(a) Find all Pareto-optimal strategies

(b) Solve the game as a TU game.

20 points (c) Find a λ-transfer solution assuming it is an NTU game.

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(5) Consider a weighted majority game with four players with the weights $5, 10, 10, 22$.

(a) Compute the Shapley-Shubik power index.

20 points (b) Find the Nucleolus.

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Total Marks = 100 points