Complex Analysis

Assignment 5, due November 12 at noon

Problem 1 of 5. Let C be a circle, and z_1, z_2 be two complex numbers, $z_1 \notin C$, $z_2 \notin C$. Show that there exists a unique Möbius map S such that $S(C) = C$, $S(z_1) = z_2$, $S'(z_1) > 0$.

Problem 2 of 5.

(1) Problem 6, page 83 of *Ahlfors*.

(2) Problem 7, page 83 of *Ahlfors*.

(3) Find all the circles which are orthogonal to both of the circles $|z| = 1$ and $|z - 1/4| = 1/4$.

Problem 3 of 5.

(1) Exercise 3, page 88 of *Ahlfors*. Assume that S is not the identity map and that there exists an integer $n \geq 2$ such that

$$S^n(z) = z \qquad \text{for every } z \in \widehat{\mathbb{C}}.$$

Here S^n denotes the n-fold composition of S with itself, and the same n must work for every z.

(2) Exercise 4, page 88 of *Ahlfors*.

Problem 4 of 5. Exercise 4, page 108 of *Ahlfors*.

Problem 5 of 5. Let f be a continuous function in a disk $B(z_0, r)$, and, for $0 < \delta < r$, let γ^δ be the circle $\{|z - z_0| = \delta\}$ oriented counterclockwise. Prove that

$$\lim_{\delta \to 0} \oint_{\gamma^\delta} \frac{f(z)}{z - z_0}\, dz = 2\pi i f(z_0).$$