Erratum to: Conjectures and results on modular representations of $\mathrm{GL}_n(K)$ for a p-adic field K

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We make a handful of corrections to the paper [BHH+25]. The third issue was found by Changjiang Du, for which we thank him heartily. We tried to fill the gap for some time but the final solution was given by ChatGPT 5.6 Sol. The first two issues were then identified and fixed with the help of ChatGPT 5.6 Sol. We now give our own accounts of how these mistakes can be fixed, keeping the notation of [BHH+25].

1 Proof of [BHH+25, Lemma 2.24]

The subtorus $Z'_{M_{Q_H}}$ of $Z_{M_{Q_H}}$ cannot be arbitrary, but we should demand that if $\sum_{\alpha\in R_H^+\backslash R(Q_H)^+} n_\alpha\alpha$ is trivial on $Z'_{M_{Q_H}}$ for some $n_\alpha \in \mathbb{Z}_{\geq 0}$, then $n_\alpha = 0$ for all $\alpha \in R_H^+\backslash R(Q_H)^+$. This condition guarantees that the subspace on the left of the displayed equation of the cited proof is indeed preserved by N_{Q_H}, and is satisfied in case that $Z'_{M_{Q_H}} = Z_{M_P}$, diagonally embedded in $H := G^{\mathrm{Gal}(K/\mathbb{Q}_p)}$.

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2 Statement and proof of [BHH$^+$25, Proposition 2.62]

The statement of [BHH$^+$25, Proposition 2.62] needs to be strengthened by demanding that *all* the irreducible constituents of $\overline{\rho}^{\mathrm{ss}}$ are pairwise non-isomorphic. For convenience we write $G \times_{\mathbb{Z}} \mathbb{F} = \mathrm{GL}(V)$ for some n-dimensional $\mathbb{F}$-vector space V. Decompose $V = V_1 \oplus \cdots \oplus V_r$ corresponding to the standard parabolic subgroup $P_{\overline{\rho}}$, and correspondingly write $\{1, \dots, n\} = T_1 \sqcup \cdots \sqcup T_r$ with $T_i := \{(\sum_{k<i} n_k) + 1, \dots, \sum_{k \le i} n_k\}$. Then the $W(P_{\overline{\rho}})$-orbits on $R^+ \setminus R(P_{\overline{\rho}})^+$ are given by $\mathcal{O}_{ij} := \{e_k - e_\ell : k \in T_i, \ell \in T_j\}$ for $i < j$.

We first observe that we can identify closed subsets of R^+ relative to $P_{\overline{\rho}}$ with partial orders of the set $\{1, \dots, r\}$ that are coarser than the total ordering induced by $\mathbb{Z}$. Indeed, by [BHH$^+$25, Def. 2.54] a closed subset $X \subseteq R^+$ relative to $P_{\overline{\rho}}$ is the same as a closed $W(P_{\overline{\rho}})$-stable subset of $R^+ \setminus R(P_{\overline{\rho}})^+$, by sending X to $X \setminus R(P_{\overline{\rho}})^+$. Thus $X \setminus R(P_{\overline{\rho}})^+ = \coprod_{(i,j) \in Z} \mathcal{O}_{ij}$ for some subset $Z \subseteq \{(i,j) : 1 \le i < j \le r\}$. Observe that $X \setminus R(P_{\overline{\rho}})^+$ is closed if and only if Z is transitive, i.e. $(i,j), (j,k) \in Z$ implies $(i,k) \in Z$.

For $1 \le i < j \le r$ define the unipotent subgroup $N_{ij} := N_{\mathcal{O}_{ij}}$ of $N_{P_{\overline{\rho}}}$. As a consequence of the commutation relation [BHH$^+$25, (2.59)] we note that

$$[N_{ij}, N_{k\ell}] \begin{cases} \subseteq N_{i\ell} & \text{if } j = k, \\ \subseteq N_{kj} & \text{if } i = \ell, \\ = 1 & \text{otherwise.} \end{cases} \tag{1}$$

We first provide a more useful version of [BHH$^+$25, Lemma 2.61].

Lemma 1. *Let $\overline{\rho} : \mathrm{Gal}(\overline{\mathbb{Q}}_p/K) \to P_{\overline{\rho}}(\mathbb{F})$ and $X_{\overline{\rho}}$ as below [BHH$^+$25, (2.60)], and assume that the irreducible constituents of $\overline{\rho}^{\mathrm{ss}}$ are pairwise non-isomorphic. Let $1 \le i < j \le r$ such that $\mathcal{O}_{ij} \subseteq R^+ \setminus X_{\overline{\rho}}$ and let $n \in N_{ij}(\mathbb{F}) \setminus \{1\}$. Then*

$$X_{n\overline{\rho}n^{-1}} = \langle X_{\overline{\rho}}, \mathcal{O}_{ij} \rangle,$$

where the right-hand side denotes the smallest closed subset relative to $P_{\overline{\rho}}$ that contains $X_{\overline{\rho}}$ and $\mathcal{O}_{ij}$.

Proof. Let $Y := \langle X_{\overline{\rho}}, \mathcal{O}_{ij} \rangle$ and $Y_0 := Y \setminus \mathcal{O}_{ij}$, so $X_{\overline{\rho}} \subseteq Y_0 \subseteq Y$. Then by the above identification of closed subsets relative to $P_{\overline{\rho}}$ with certain partial orders, it follows easily that Y_0 is also closed relative to $P_{\overline{\rho}}$. We deduce that $N_{Y_0 \setminus R(P_{\overline{\rho}})^+} \lhd N_{Y \setminus R(P_{\overline{\rho}})^+}$ because N_{ij} normalizes $N_{Y_0 \setminus R(P_{\overline{\rho}})^+}$ by (1). As $\overline{\rho}$ takes values in $M(\mathbb{F}) N_{X_{\overline{\rho}}}(\mathbb{F})$ and $n \in N_{ij}(\mathbb{F})$ we deduce that $n\overline{\rho}n^{-1}$ takes values in $M(\mathbb{F}) N_Y(\mathbb{F})$, i.e. $X_{n\overline{\rho}n^{-1}} \subseteq \langle X_{\overline{\rho}}, \mathcal{O}_{ij} \rangle$.

We show that $\mathcal{O}_{ij} \subseteq X_{n\overline{\rho}n^{-1}}$. Write $\overline{\rho}(g) = \overline{\rho}^{\mathrm{ss}}(g) n(g)$ with $n(g) \in N_{X_{\overline{\rho}} \setminus R(P_{\overline{\rho}})^+}(\mathbb{F}) \subseteq N_{Y_0 \setminus R(P_{\overline{\rho}})^+}(\mathbb{F})$. (We write for short $\overline{\rho}^{\mathrm{ss}}$ for $\overline{\rho}^{P_{\overline{\rho}}-\mathrm{ss}}$.) Then

$$n\overline{\rho}(g)n^{-1} \in n\overline{\rho}^{\mathrm{ss}}(g)n^{-1} \cdot N_{Y_0 \setminus R(P_{\overline{\rho}})^+}(\mathbb{F}), \tag{2}$$

as $N_{Y_0\setminus R(P_{\overline{\rho}})^+} \lhd N_{Y\setminus R(P_{\overline{\rho}})^+}$. Write $n = 1 + a$, where $a \in \mathrm{End}(V)$ sends V_j to V_i and is zero on $V_{j'}$ for $j' \neq j$. Hence $a^2 = 0$ and

$$\begin{aligned} n\overline{\rho}^{\mathrm{ss}}(g)n^{-1} &= (1+a)\overline{\rho}^{\mathrm{ss}}(g)(1-a) = \overline{\rho}^{\mathrm{ss}}(g) + a\overline{\rho}^{\mathrm{ss}}(g) - \overline{\rho}^{\mathrm{ss}}(g)a \\ &= \overline{\rho}^{\mathrm{ss}}(g)\Big(1 + \overline{\rho}^{\mathrm{ss}}(g)^{-1}a\overline{\rho}^{\mathrm{ss}}(g) - a\Big) \in \overline{\rho}^{\mathrm{ss}}(g)N_{ij}(\mathbb{F}). \end{aligned} \tag{3}$$

As $a \neq 0$ (since $n \neq 1$) we deduce that $\overline{\rho}^{\mathrm{ss}}(g)^{-1}a\overline{\rho}^{\mathrm{ss}}(g) - a \neq 0$ for some $g \in \mathrm{Gal}(\overline{\mathbb{Q}}_p/K)$, because $V_i \ncong V_j$ as $\mathrm{Gal}(\overline{\mathbb{Q}}_p/K)$-representations by assumption. Combining this with (2) and (3) we deduce that $\mathcal{O}_{ij} \subseteq X_{n\overline{\rho}n^{-1}}$.

It remains to show that $X_{\overline{\rho}} \subseteq X_{n\overline{\rho}n^{-1}} =: X'$. As $n\overline{\rho}n^{-1}$ takes values in $M(\mathbb{F})N_{X'}(\mathbb{F})$ and n takes values in $N_{X'}(\mathbb{F})$ by the preceding paragraph, we deduce that $\overline{\rho}$ takes values in $M(\mathbb{F})N_{X'}(\mathbb{F})$, so that $X_{\overline{\rho}} \subseteq X'$ by definition of $X_{\overline{\rho}}$. □

We now fix [BHH+25, Proposition 2.62].

Proposition 2. *Let $\overline{\rho} : \mathrm{Gal}(\overline{\mathbb{Q}}_p/K) \to P_{\overline{\rho}}(\mathbb{F})$ and $X_{\overline{\rho}}$ as below [BHH+25, (2.60)], and assume that the irreducible constituents of $\overline{\rho}^{\mathrm{ss}}$ are pairwise non-isomorphic. Then there is $h_0 \in P_{\overline{\rho}}(\mathbb{F})$ (non-unique in general) such that $X_{h_0\overline{\rho}h_0^{-1}} \subseteq X_{h\overline{\rho}h^{-1}}$ for all $h \in P_{\overline{\rho}}(\mathbb{F})$.*

Proof. As in the proof of [BHH+25, Proposition 2.62], we may assume that $X_{h\overline{\rho}h^{-1}} = X_{\overline{\rho}}$ for all $h \in N_{X_{\overline{\rho}}\setminus R(P_{\overline{\rho}})^+}(\mathbb{F})$, and it is enough to prove $X_{\overline{\rho}} \subseteq X_{h\overline{\rho}h^{-1}}$ for all $h \in N_{P_{\overline{\rho}}}(\mathbb{F})$. By [BHH+25, Lemma 2.59] (see also the first paragraph of the proof) we may write $R^+ = X_{\overline{\rho}} \sqcup \mathcal{O}_1 \sqcup \cdots \sqcup \mathcal{O}_m$, where each $\mathcal{O}_i$ is a $W(P_{\overline{\rho}})$-orbit, such that $X_{\overline{\rho},i} := X_{\overline{\rho}} \sqcup \coprod_{k\leq i} \mathcal{O}_k$ is closed relative to $P_{\overline{\rho}}$ for all $0 \leq i \leq m$. Factor $h \in N_{P_{\overline{\rho}}}(\mathbb{F})$ as $h = h_m h_{m-1} \cdots h_1 h_{\overline{\rho}}$ such that $h_i \in N_{\mathcal{O}_i}(\mathbb{F})$ and $h_{\overline{\rho}} \in N_{X_{\overline{\rho}}\setminus R(P_{\overline{\rho}})^+}(\mathbb{F})$. Let $\overline{\rho}_i := h_i \cdots h_1 h_{\overline{\rho}}\overline{\rho}h_{\overline{\rho}}^{-1}h_1^{-1}\cdots h_i^{-1}$ for $1 \leq i \leq m$ and $\overline{\rho}_0 := h_{\overline{\rho}}\overline{\rho}h_{\overline{\rho}}^{-1}$.

We show by induction that $X_{\overline{\rho}} \subseteq X_{\overline{\rho}_i} \subseteq X_{\overline{\rho},i}$ for all i, hence the proposition follows by setting $i = m$. For $i = 0$ the claim holds by the first sentence of this proof. Assume the claim holds for $i - 1$ (some $i > 0$). If $h_i = 1$, then $\overline{\rho}_i = \overline{\rho}_{i-1}$ and we are done. If $h_i \neq 1$, then, as $\mathcal{O}_i \subseteq R^+ \setminus X_{\overline{\rho}_{i-1}}$ by induction, we have by Lemma 1 that $X_{\overline{\rho}_i} = \langle X_{\overline{\rho}_{i-1}}, \mathcal{O}_i\rangle$, and the claim follows. □

All occurrences of [BHH+25, Proposition 2.62] should be replaced by Proposition 2 above, and therefore in the statements of [BHH+25, Definition 2.63] and [BHH+25, Theorem 2.65] the condition "of dimension 1" should be removed.

3 Proof of [BHH+25, Theorem 3.29]

In this proof an argument is incomplete. We claimed that a surjection of $\mathbb{F}[\![N_0]\!]_S$-modules $(\pi^\vee)_S \twoheadrightarrow M^\vee[X^{-1}]$ is continuous, but the argument is far from being sufficient.

If (D, φ_D) is an étale φ-module over $\mathbb{F}(\!(X)\!)$, we denote by ψ_D the $\mathbb{F}$-linear endomorphism defined, for instance, in [Col10, §II.3.1]. It is easily checked from the definition that

$$\forall x \in D, \quad x = \sum_{i=0}^{p-1} (1+X)^i \varphi_D(\psi_D((1+X)^{-i}x)). \tag{4}$$

Lemma 3. *Let (D, φ_D) be an étale φ-module over $\mathbb{F}(\!(X)\!)$. Let $\Lambda \subseteq D$ be an open finitely generated $\mathbb{F}[\![X]\!]$-submodule. Let $a \in \mathbb{Z}$ such that $\varphi_D(\Lambda) \subseteq X^{-a}\Lambda$. If $L \subseteq D$ is an $\mathbb{F}[\![X]\!]$-submodule and n is an integer such that $\psi_D(L) \subseteq X^n\Lambda$, then $L \subseteq X^{pn-a}\Lambda$.*

Proof. This is a direct consequence of formula (4), of the fact that $(1+X) \in \mathbb{F}[\![X]\!]^\times$ and of the inclusion

$$\varphi_D(\psi_D((1+X)^{-i}L)) \subseteq \varphi_D(X^n\Lambda) \subseteq X^{np}\varphi_D(\Lambda) \subseteq X^{np-a}\Lambda. \qquad \square$$

Lemma 4. *Let $M \subseteq \pi^{N_1}$ be a finitely generated $\mathbb{F}[\![X]\!][F]$-submodule that is admissible as an $\mathbb{F}[\![X]\!]$-module and satisfies $M/XM = 0$. Let $q : \pi^\vee \twoheadrightarrow M^\vee$ be the $\mathbb{F}$-linear dual map of the inclusion $M \subseteq \pi$. There exists an integer $c \geq 0$ such that*

$$\forall n \geq 0, \quad q(\mathfrak{m}_{I_1}^{n+c}\pi^\vee) \subseteq X^n M^\vee. \tag{5}$$

Proof. As $M/XM = 0$, we have $M^\vee[X] = 0$ so that $M^\vee$ is a finite free $\mathbb{F}[\![X]\!]$-module. Recall that, in [BHH+25, §2.1.1], we endow $D \overset{\text{def}}{=} M^\vee[X^{-1}]$ with an additional structure of étale φ-module in such a way that ψ_D restricts to $F^\vee$ on $M^\vee$ (Indeed, equation (4) says that $\varphi_D^*\big(\sum_{i=0}^{p-1}(1+X)^i \otimes \psi_D((1+X)^{-i}f)\big) = f$ for all $f \in M^\vee$ and by construction of φ_D^* in [BHH+25, § 2.1.1] and [BHH+25, (2.6)] we also have

$$\varphi_D^*\Big(\sum_{i=0}^{p-1}(1+X)^i \otimes \Big((\mathrm{Id} \otimes F)^\vee(f)\big((1+X)^{-i} \otimes \cdot\big)\Big)\Big) = f.$$

By bijectivity of φ_D^* we have

$$\sum_{i=0}^{p-1}(1+X)^i \otimes \psi_D((1+X)^{-i}f) = \sum_{i=0}^{p-1}(1+X)^i \otimes \Big((\mathrm{Id} \otimes F)^\vee(f)\big((1+X)^{-i} \otimes \cdot\big)\Big)$$

and we conclude from [BHH+25, (2.6)] that $(\mathrm{Id} \otimes F)^\vee(f)$ is the element of $(F[\![X]\!] \otimes_\varphi M)^\vee$ that sends $(1+X)^{-i} \otimes m$ to $\psi_D((1+X)^{-i}f)(m)$ for $0 \leq i \leq p-1$ and $m \in M$. Setting $i = 0$ we deduce that $F^\vee(f) = \psi_D(f)$.) We fix an integer $a \geq 0$ such that $\varphi_D(M^\vee) \subseteq X^{-a}M^\vee$. Let $b \overset{\text{def}}{=} \lceil \frac{a+p}{p-1} \rceil \geq 1$.

We deduce from [BHH+25, Lemma 3.28] that the map $q : \pi^\vee \twoheadrightarrow M^\vee$ is continuous when $\pi^\vee$ is endowed with the $\mathfrak{m}_{I_1}$-adic topology and $M^\vee$ the X-adic topology. Therefore there exists an integer $c \geq 0$ such that, for $n \geq c$, $q(\mathfrak{m}_{I_1}^n\pi^\vee) \subseteq X^{b-1}M^\vee$.

Now we check by induction on $n \geq 0$ the inclusion (5). If $0 \leq n < b$, we have

$$q(\mathfrak{m}_{I_1}^{n+c}\pi^\vee) \subseteq X^{b-1}M^\vee \subseteq X^n M^\vee,$$

so that (5) is true for $n < b$. Let $k \geq b$. We assume that (5) is true for $n < k$ and show that it is true for $n = k$. Let $s, t \geq 0$ be integers such that $2s + t = k + c$. It follows from formula [BHH+25, (3.15)] with $r = 0$ that

$$\psi(\mathbb{F}[\![N_0]\!]\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq \mathfrak{m}_{I_1}^{k+c}\pi^\vee$$

(we recall that ψ was defined in [BHH+25, (3.10)]). For each $u \in N_1/N_1^p$, we choose a lift $\tilde{u} \in N_1$ of u. From the definition of F on π^{N_1}, we have, for $x \in \pi^\vee$,

$$q\left(\sum_{u \in N_1/N_1^p} \psi(\delta_{\tilde{u}} x)\right) = F^\vee(q(x)) = \psi_D(q(x)).$$

It follows that

$$\psi_D(q(\mathbb{F}[\![N_0]\!]\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee)) \subseteq q(\psi(\mathbb{F}[\![N_0]\!]\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee)) \subseteq q(\mathfrak{m}_{I_1}^{k+c}\pi^\vee) \subseteq q(\mathfrak{m}_{I_1}^{k-1+c}\pi^\vee) \subseteq X^{k-1}M^\vee,$$

where the last inclusion is deduced from our induction hypothesis. By Lemma 3 applied with $\Lambda = M^\vee$ and $L = q(\mathbb{F}[\![N_0]\!]\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee)$ (note that q is $\mathbb{F}[\![N_0]\!]$-linear so that $q(\mathbb{F}[\![N_0]\!]\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee)$ is an $\mathbb{F}[\![X]\!]$-submodule of D), we have

$$q(\mathbb{F}[\![N_0]\!]\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq X^{p(k-1)-a}M^\vee.$$

As $k \geq b$, we have $p(k-1) - a \geq k$, which gives us the inclusion

$$q(\mathbb{F}[\![N_0]\!]\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq X^k M^\vee. \tag{6}$$

Now choose integers $r, s, t \geq 0$ such that $r + 2s + t = k + c$. As the map q is linear with respect to the local homomorphism of local rings $\mathbb{F}[\![N_0]\!] \twoheadrightarrow \mathbb{F}[\![X]\!]$ induced by the trace $\mathrm{tr} : \mathcal{O}_K \to \mathbb{Z}_p$ (which sends $\mathfrak{m}_{N_0}$ onto $X\mathbb{F}[\![X]\!]$), we have

$$q(\mathfrak{m}_{N_0}^r \mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq X^r q(\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee).$$

If $r \geq k$, we have, $q(\mathfrak{m}_{N_0}^r \mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq X^k M^\vee$, whereas if $r \leq k$ we have $2s + t \geq c$ and we can apply our induction hypothesis (if $2s+t < k+c$) and (6) (if $2s+t = k+c$) to deduce

$$q(\mathfrak{m}_{N_0}^r \mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq X^r q(\mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq X^{r+2s+t-c}M^\vee = X^k M^\vee.$$

(We recall [BHH+25, (3.12)] for the second inclusion.) Finally we can conclude the induction as

$$q(\mathfrak{m}_{I_1}^{k+c}\pi^\vee) \subseteq \sum_{\substack{r,s,t \geq 0 \\ r+2s+t=k+c}} q(\mathfrak{m}_{N_0}^r \mathfrak{m}_{T_0}^s \mathfrak{m}_{N_0^-}^t \pi^\vee) \subseteq X^k M^\vee. \qquad \square$$

We have to modify the proof of [BHH$^+$25, Thm 3.29] as follows. The sentence "By definition of the tensor product filtration ..." should be replaced by the following reasoning. Let $M' \subseteq M$ be the $\mathbb{F}[\![X]\!]$-submodule of M such that $(M')^\vee$ is identified with the quotient of $M^\vee$ by its torsion submodule. Then M' is stable by F (as $(M')^\vee$ is stable by $F^\vee$ according to the formula $XF^\vee(\cdot) = F^\vee(X^p\cdot)$) and we have $M^\vee[X^{-1}] = (M')^\vee[X^{-1}]$. Moreover M' is finitely generated over $\mathbb{F}[\![X]\!][F]$ by [Bre15, Lemme 5.2] (see also [Eme, Prop. 3.3]). Thus we can apply Lemma 4 to M' to deduce that there exists an integer $c \geq 0$ such that $q(\mathfrak{m}_{I_1}^n \pi^\vee) \subseteq X^{n-c}(M')^\vee$ for all $n \geq c$. This implies, together with [BHH$^+$25, Lemmas 3.38 & 3.40], that, for any $n, k \geq 0$ such that $n + kf \geq c$,

$$q((Y_0 \cdots Y_{f-1})^{-k}\mathfrak{m}_{I_1}^{n+kf}\pi^\vee) = X^{-kf}q(\mathfrak{m}_{I_1}^{n+kf}\pi^\vee) \subseteq X^{-kf+n+kf-c}(M')^\vee = X^{n-c}(M')^\vee.$$

Therefore we have $q(F_n\pi_S^\vee) \subseteq X^{-n-c}(M')^\vee$ for all $n \leq 0$, which implies that the map $\pi_S^\vee \to M^\vee[X^{-1}] = (M')^\vee[X^{-1}]$ is continuous.

4 Other minor corrections in [BHH$^+$25]

- In the definition of parabolic induction preceding Lemma 2.5, it should say $x \in G(K)$.
- In the proof of Corollary 3.9, $T = X$.
- Six lines before equation (3.16), it should say $(a, m) \in A \times D_A(\pi)$.
- In Remark 3.22, $\gamma_{-1} = \gamma$.
- In Remark 3.26, third line, the words "the image of" should be removed.
- There is an unfortunate mistake in numbering in section 2.5 that was introduced during copy-editing. In that section, Conjecture 2.1 should be Conjecture 2.86, Remark 2.2 should be Remark 2.87, etc.

References

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