5c) $V=\mathbb{R}^3$, $W=\left\{\begin{pmatrix}x\\y\\z\end{pmatrix}\in V : x^2+y^2-z^2=0\right\}$

Is W a subspace of V?

- $\vec{0}\in W$ $(0^2+0^2-0^2=0)$

1 • "W is closed under scalar mult." (*) $x^2+y^2-z^2-(2x)^2-(2y)^2+(2z)^2$

- "The vector, when multiplied by a scalar, still belongs to the same v.s."
  ↳ needs some more precision.
- "For any vector $\vec{v},\vec{w}\in W$, for any scalar λ,μ, $\lambda\vec{v}+\mu\vec{w}\in W$"
  ↳ "W is closed under linear combinations."
- "For any vector $\vec{v}\in W$, for any scalar λ, $\lambda\cdot\vec{v}\in W$." (*)

Proof of 1 Let $\vec{v}=\begin{pmatrix}x\\y\\z\end{pmatrix}$ st. $x^2+y^2-z^2=0$. (i.e. $\vec{v}\in W$). and λ a scalar.

$$\lambda\cdot\vec{v}=\begin{pmatrix}\lambda x\\\lambda y\\\lambda z\end{pmatrix} \rightarrow (\lambda x)^2+(\lambda y)^2-(\lambda z)^2 \overset{?}{=} 0$$
$$=\lambda^2x^2+\lambda^2y^2-\lambda^2z^2$$
$$=\lambda^2\underbrace{(x^2+y^2-z^2)}_{=0}$$
$$=\lambda^2\cdot 0=0$$

- Is W closed under addition? 2

Let $\vec{v}_1=\begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$ st. $x_1^2+y_1^2-z_1^2=0$ and $\vec{v}_2=\begin{pmatrix}x_2\\y_2\\z_2\end{pmatrix}$ st. $x_2^2+y_2^2-z_2^2=0$.

$$\vec{v}_1+\vec{v}_2=\begin{pmatrix}x_1+x_2\\y_1+y_2\\z_1+z_2\end{pmatrix}$$

$\rightsquigarrow (x_1+x_2)^2+(y_1+y_2)^2-(z_1+z_2)^2=$ [expand] = [simplify]

$$=2x_1x_2+2y_1y_2+2z_1z_2 \overset{?}{\neq} 0$$

Choose

$$\vec{v}_1=\vec{v}_2=\begin{pmatrix}3\\4\\5\end{pmatrix} \rightsquigarrow 2\cdot3\cdot3+2\cdot4\cdot4+2\cdot5\cdot5\neq 0$$

$(9+16=25)$

W is not closed under addition! $\Rightarrow$ W is not a subspace of V.