MAT157Y PROBLEM SET 12

– DUE JANUARY 27

Please make an effort to write clearly and concisely.

1. Given constants a, $b \in \mathbb{R}$, find constants C, D so that $a\sin x + b\cos x = C\cos(x + D)$.

2. Prove $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$.

3. Evaluate $\lim_{x\to 0}\Big(\frac{1}{x} - \cot x\Big)$

4. Find the "double-angle formulas" for $\cos(2x)$ and $\sin(2x)$ in terms of $\cos x$ and $\sin x$, and use them to find the "half-angle formulas" for $\sin\frac{x}{2}$ and $\cos\frac{x}{2}$.

5. Suppose $t = \tan\frac{\theta}{2}$. Express $\cos\theta$ and $\sin\theta$ in terms of t.

6. Evaluate $\lim_{x\to 0}\Big(\frac{x^2}{1-\cos x}\Big)$.

7. For positive integers m, n, prove that

$$\sin(mx)\sin(nx) = \frac{1}{2}\Big(\cos\big((m-n)x\big) - \cos\big((m+n)x\big)\Big)$$

and use the result to show that

$$\int_0^{2\pi} \sin(mx)\sin(nx)\,dx = \begin{cases} 0, & \text{if} \quad m \neq n \\ \pi, & \text{if} \quad m = n. \end{cases}$$

Then evaluate

$$\int_0^{2\pi} \cos(mx)\cos(nx)\,dx \quad \text{and} \quad \int_0^{2\pi} \cos(mx)\sin(nx)\,dx.$$

8*. Just for fun, if you have a calculator or software that draws graphs, plot $y = \frac{\sin 7x}{\sin x}$, for $x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$. Then try $y = \frac{\sin 17x}{\sin x}$.