Problem 1. Given $n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}$, let $f_n(x) = |x|^n$. For which values of n is f_n differentiable at 0?

Problem 2 (Spivak 9-27)**.** Let $S_n(x) = x^n$ and $0 \leq k \leq n$. Prove that

$$S_n^{(k)}(x) = \frac{n!}{(n-k)!} x^{n-k} \tag{1}$$

Problem 3. Prove the quotient rule for derivatives using the product rule and chain rule:

$$\big(f/g\big)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2} \tag{2}$$

Problem 4 (Inspired from Spivak 9-22)**.**

1. Suppose f is differentiable at x. Show that

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} \tag{3}$$

2. Does the converse hold? That is, if the above limit exists and equals L, is f differentiable at x with derivative $f'(x) = L$? Prove this is true or find a counterexample.

Problem 5. Without looking at your notes, prove the chain rule.

Problem 6.

1. Suppose $f(c) = b$ and f is differentiable at c with $f'(c) \neq 0$. Compute the derivative of f^{-1} at b.

2. Use the previous part to compute $\frac{\partial}{\partial x} \arcsin(x)\big|_{x=0}$ (recall arcsin is the inverse function of sin).

Problem 7. Let $f\colon [a,b] \to \mathbb{R}$ be continuous differentiable on (a,b). Come up with a definition of one-sided derivatives; that is, what should be meant by $f'(a)$ and $f'(b)$ (or, if you prefer $f'(a^+)$ and $f'(b^-)$)?

Problem 8. A function $f\colon (a,b) \to \mathbb{R}$ is called Lipschitz if there exists a $k > 0$ such that for all x, $y \in [a,b]$,

$$\big|f(x) - f(y)\big| \leq k|x - y| \tag{4}$$

Prove that if f is differentiable on $[a,b]$ (see problem 7) and f' is continuous on $[a,b]$, then f is Lipschitz.