

Problem 1. Differentiate the following functions of x :

1. $f_1(x) = x^{\log y} - y^{\log x}$ where $y > 0$.

2. $f_2(x) = x^{x^x}$

3. $f_3(x) = (\log x)^x$

4. $f_4(x) = e^{-\frac{x^3}{3}}$

5. $f_5(x) = e^{e^{e^x}}$

Problem 2 (Spivak 18-2). For each instance of f , compute the derivative of $\log \circ f$.

1. $f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}$

2. $f(x) = \frac{(3-x)^{1/3}x^2}{(1-x)(3+x)^{2/3}}$

3. $f(x) = \prod_{k=1}^n f_k(x)$ for differentiable functions f_1, f_2, \dots, f_n .

What happens when you multiply your result by $f(x)$?

Problem 3 (Spivak 19-3). Express the following antiderivatives as elementary functions:

1. $\int x^3 e^{x^2} dx$

2. (*) $\int \sec^3 x dx$

Problem 4 (*). Let $I_n = \int \sin^n(x) dx$. Express I_n in terms of I_1, I_2, \dots, I_{n-1} . Use this recursion formula to compute $\int \sin^5(x) dx$.