1 Comments from problem sets

- An ε-δ *proof* means that you *directly* use the definition of a limit and do not appeal to theorems. Using the product rule to simplify your problem is not the focus of such a problem. The purpose of these questions is to practice working with the definition of a limit directly. The theorems you learn in class are a (useful) shortcut that is better understood when you have spent some time working with the raw definition.
- In every problem you work on, *define all the variables you are working with*. This includes ε and δ. Getting into the habit of writing unambiguously will make your writing more understandable.

2 Practice problems

Problem 1. Consider a bounded function $f\colon [a,b] \to \mathbb{R}$ and a partition $P = \{t_0, t_1, \dots, t_n\}$ such that

$$a = t_0 < t_1 < \cdots < t_n = b \tag{1}$$

Without looking at your notes, define the upper sum, $\mathcal{U}(f,P)$, and the lower sum, $\mathcal{L}(f,P)$, of f with respect to P.

Problem 2. Let $f\colon [a,b] \to \mathbb{R}$ satisfy $f(x) > 0$ for all $x \in [a,b]$. Prove that for some partition P, $\mathcal{L}(f,P) > 0$.

Problem 3. Consider the function

$$\begin{aligned} f\colon [0,1] &\to \mathbb{R} \\ x &\mapsto \begin{cases} \frac{1}{q} & \text{if } x \in \mathbb{Q} \text{ and } x = \frac{p}{q} \text{ in lowest terms} \\ 0 & \text{else} \end{cases} \end{aligned} \tag{2}$$

Denote by P_n the uniform partition of $[0,1]$ into n intervals. That is,

$$P_n = \left\{0, \frac{1}{n}, \frac{2}{n}, \dots, 1\right\} \tag{3}$$

Compute $\lim_{n\to\infty} \mathcal{U}(f,P_n)$ and $\mathcal{L}(f,P_n)$.