## General Information.

- The Term Test will take place during your scheduled lecture section on Tuesday or Wednesday in Week 8 (March 3 or March 4, depending on when your lecture is scheduled), and will cover material from Chapters 1 through 6 of the course lecture notes.
- Details about the time and location of your sitting can be found on page 2 of this document.
- This document contains a practice test (starting on page 3 of the pdf). **While the problems are different than what will appear on the actual term test**, the instructions and format are identical. The term test will have precisely the same number of problems included on the practice test.
- Please keep in mind that reviewing this practice test is not enough to prepare you for the actual test. The practice test is only meant to give you an example of the style and format of the assessment.
- Solutions to the practice test will be posted by the end of the day on **Thursday, February 26th**.

## Tips for Success

Here is a (non-exhaustive) list of tips that might help you prepare for the term test.

1. Start early, and spread your studying out over several days.
2. Study all definitions and proofs from the Chapter Activity Packets (CAP1-6).
3. Make flashcards for definitions. Consider using a Frayer model.
4. Use active recall techniques when studying proofs from the CAPs.
5. Complete the attached practice test on your own without help. Only ask for help after you've given yourself time to get stuck and have attempted to solve all problems on your own.
6. Redo material from the course (including Lecture Activities from the CAPs, Webwork, and Chapter Exercises). Don't just "review" or "look over" what we've done. **Rework problems without the solutions**. Only use the solutions to check your work.
7. Carefully review the feedback you were given on all Problem Set Quizzes.
8. Solve the remaining chapter exercises problems that were not assigned on the problem sets.
9. Use the additional resources list in the preface of the course lecture notes (located on page 6 of the pdf) to find new problems to solve.
10. Form a study group. Write new problems and exchange them with your classmates for extra practice.
11. Don't write anything down that you do not completely understand. Be stubborn. If you can't understand something after giving it a good effort, ask for help.
12. Attend office hours. Challenge yourself to ask questions and to let the instructor know when something doesn't make sense.
13. Attend your tutorial section on Monday before the test for further review.

### Term Test Times and Locations

Note that you must attend the term test sitting for the **lecture section you are registered for**. Some sections will be split into two rooms based on the **first letter of student surname** (last name), as indicated in the table below.

| **Lecture Section** | **Test Date** | **Test Time** | **Surname** | **Location** | **Building Name** |
|---|---|---|---|---|---|
| LEC0101 | Tues, March 3 | 2:10 - 4pm | A - Z | EX 200 | Exam Centre |
| LEC5101 | Tues, March 3 | 6:10 - 8pm | A - Z | EX 100 | Exam Centre |
| LEC0201 | Wed, March 4 | 9:10 - 11am | A - LEE | EX 310 | Exam Centre |
| LEC0201 | Wed, March 4 | 9:10 - 11am | LI - Z | MP 203 | McLennan Physical Labs |
| LEC0301 | Wed, March 4 | 11:10am - 1pm | A - MENG | EX 310 | Exam Centre |
| LEC0301 | Wed, March 4 | 11:10am - 1pm | MI - Z | MB 128 | Mining Building |
| LEC0401 | Wed, March 4 | 1:10 - 3pm | A - Z | EX 100 | Exam Centre |
| LEC0501 | Wed, March 4 | 3:10 - 5pm | A - LY | EX 310 | Exam Centre |
| LEC0501 | Wed, March 4 | 3:10 - 5pm | M - Z | MP 203 | McLennan Physical Labs |

For example, say that I'm registered for LEC0301. Then, my term test will take place on Wednesday, March 4, from 11:10am-1pm. Since my name is Elisa **B**ellah, my term test will take place in EX 310. If instead my name was Elisa **N**ellah, my term test would take place in MB 128.

If you are unsure about the time or location of your term test sitting, please either ask your course instructor, or email the administrative inbox. Note that there will be **no makeup** term tests under any circumstance. If you need to miss the term test, you will receive a zero on the assignment, and we will replace your term test score with your final exam score (as stated on the course syllabus).

**University of Toronto**
**Faculty of Arts and Sciences**
**PRACTICE MAT223H1F Term Test**

**Winter 2026**
**Duration: 110 minutes**
**Aids Allowed: None**

Name (First then Last): ______________________________________________

University Email Address: ________________________________@mail.utoronto.ca

Student Number: ______________________________________________

General Instructions:

- Fill out your name, student number, and email address at the top of this page.
- This test contains three sections:
  - **Section A** (8 points available) includes definition statements and theorem proofs.
  - **Section B** (15 points available) includes computational and multiple choice problems.
  - **Section C** (15 points available) includes conceptual and proof-based problems.

  Please read the instructions at the beginning of each section carefully.
- No calculators, notes, or electronics are permitted. Turn off and place all cell phones, smart watches, electronic devices, and unauthorized study materials in your bag under your desk. These devices may not be left in your pockets.
- Place your TCard on your desk so that it can be seen by the invigilators.
- All work must be completed in the space provided. There is additional space at the back of this packet if needed. Do not detach these pages.
- Please ask questions if anything is unclear.
- Once you've finished working, close your exam and then raise your hand. We will verify your name against your TCard and collect your exam.
- If you are still working when time is called, promptly close the test packet and wait for an invigilator to come collect your test.

Special Instructions:

- Write legibly and darkly. If we cannot read your work, we will not grade the problem.
- Erase or cross out any work you do not wish to have scored, and clearly indicate if there is work on another page you want scored.
- Fill in your bubbles completely.

Good: ● A ○ B    Bad: ◐ A ⊙ B ⊗ C

# Section A.

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INSTRUCTIONS:

1. The problems in this section will ask you to **complete a definition** or to **prove a theorem** from the course lecture notes.
2. Definitions must be stated precisely as they are in the course lecture notes (up to rewording). Each definition statement is worth **one point** and no partial credit will be given.
3. Theorem proofs will each be worth **five points**, which will be awarded using our standard rubric (which is available in the Section C instructions).

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On test day, in this section there will be **three definitions** (each worth 1 point) and **one scaffolded proof** (worth 5 points) chosen from the Chapter Activity Packets (CAP1-CAP6). Note that these questions will be scaffolded precisely as they appear in the Chapter Activity Packets.

# Section B.

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INSTRUCTIONS:

1. Each problem in this section is worth **three points**. Problems with multiple parts will be worth one point each. Otherwise, no partial credit will be given.
2. You do not need to show your work or provide justification on any problem in Section B.
3. **Your answer must be placed in the answer box provided.**
4. We have provided extra space for your scratch work on each problem, but nothing outside of the answer box will be considered toward your score on the Section B problems.

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B1. (3 points) Consider the system of linear equations

$$\begin{cases} x + 2z = a \\ y + z = 1 \\ x + 2y + 4z = 0 \end{cases}$$

where $a \in \mathbb{R}$ is an arbitrary (fixed) real number. For which value(s) of $a$ is the system consistent?

| |
|---|
| **Answer:** $a =$ |

B2. (3 points). Determine which of the following sets are vector subspaces and which are not. For those that are, find their dimension.

a) $V = \left\{ \begin{pmatrix} 2x+2y \\ x+y \\ x+y \end{pmatrix} : x, y \geq 0 \right\}$

**Answer:** ○ Is not a vector space ○ Is a vector space with dimension

$\dim(V) =$ ☐

b) $W = \left\{ \vec{x} \in \mathbb{R}^2 : \vec{x} \text{ and } \begin{pmatrix} 1 \\ 7 \end{pmatrix} \text{ are linearly dependent} \right\}.$

**Answer:** ○ Is not a vector space ○ Is a vector space with dimension

$\dim(W) =$ ☐

c) $U = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 : x + 2y + 3z = 0 \right\}.$

**Answer:** ○ Is not a vector space ○ Is a vector space with dimension

$\dim(U) =$ ☐

B3. (3 points) Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be a linear transformation given by

$$F\left(\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}\right) = \begin{pmatrix} x_1 + x_2 \\ x_2 + x_3 + x_4 \\ x_3 + x_4 \end{pmatrix}.$$

a) Find the defining matrix for $F$.

| |
|---|
| $M_F =$ |

b) Find the nullity of $F$.

| |
|---|
| nullity$(F) =$ |

c) Is $F$ injective, surjective, or bijective?

**Answer:**

- ○ $F$ is **injective** and not surjective
- ○ $F$ is **bijective**
- ○ $F$ is **surjective** and not injective
- ○ None of the above

B4. Determine which of the following matrices are invertible. If there is not enough information to determine whether the matrix is invertible, select "not enough information".

a) $A + B$ where $A$ and $B$ are invertible matrices

| | | |
|---|---|---|
| ◯ is invertible | ◯ is not invertible | ◯ not enough information |

b) $AB$ where $A$ and $B$ are two $n \times n$ matrices and the columns of $B$ are linearly dependent.

| | | |
|---|---|---|
| ◯ is invertible | ◯ is not invertible | ◯ not enough information |

c) The matrix $A = \begin{pmatrix} T(\vec{v}_1) & \cdots & T(\vec{v}_n) \end{pmatrix}$ where $T : \mathbb{R}^n \to \mathbb{R}^n$ is an injective linear transformation and $\{\vec{v}_1, \ldots, \vec{v}_n\}$ is a basis for $\mathbb{R}^n$

| | | |
|---|---|---|
| ◯ is invertible | ◯ is not invertible | ◯ not enough information. |

B5. (3 points) Give an example of a $4 \times 5$ matrix $A$ **in reduced row echelon form** that satisfies all of the following conditions:

- $A$ has **exactly one** row of zeros
- $A$ does not have a pivot in the first column
- $A$ contains at least one entry other than 0 or 1
- The system of linear equations with $A$ as its **augmented matrix** is inconsistent.

$$A = \begin{pmatrix} \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \\ \square & \square & \square & \square & \square \end{pmatrix}$$

# Section C.

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INSTRUCTIONS:

1. Each problem in this section is worth **5 points**.
2. You must provide justification for all of your answers in Section C.
3. Points will be awarded based on the rubric below. Note that half points may be awarded, and further rubric items may be added to cover potential cases not outlined below.

| **Points** | **Rubric** |
|---|---|
| 5 | Solution is presented with clear justification that is logically complete and correct. May include minor typos and computational errors if they do not majorly impact the argument. No important steps are missing or assumed. All assumptions and special cases have been covered. All suggestions for improvement come under the category of "improvements for clarity" rather than "correcting logical errors". Omission of details will be judged depending on context of the material, with simpler steps being acceptable for omission when covering more advanced topics. |
| 4 | Solution is close to full and complete, but contains either a computational error or an error in reasoning that majorly impacts the argument. This score is also appropriate for solutions that are mathematically sound but confusingly written. |
| 3 | Solution is incorrect, but understanding of the problem was demonstrated and student provided a clear outline of a potential approach with information about where they got stuck **-or-** solution is correct but justification is insufficient or so confusingly written that it cannot be followed with a reasonable amount of effort. |
| 2 | Solution is incorrect, but student demonstrated understanding of the problem **-or-** solution is correct and student did not provide justification for their answer. |
| 1 | Solution is incorrect and student did not demonstrate understanding of the problem, but did demonstrate some knowledge of relevant material. |
| 0 | Solution is incorrect or incomplete, and there was no demonstration of knowledge of relevant material. |

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C1. (5 points) Let $\{\vec{v}_0, \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_m\}$ be a linearly independent set of vectors in $\mathbb{R}^n$. Show that the set

$$\{\vec{v}_0 + \vec{v}_1, \vec{v}_0 + \vec{v}_2, \ldots, \vec{v}_0 + \vec{v}_m\}$$

is also linearly independent.

*Proof.*

C2. (5 points) True or False: If $A$ is an $n \times m$ matrix with $n < m$, then nullity$(A) > 0$. **If true provide a proof. If false, provide a counterexample, and justify why this is a counterexample.**

○ True ○ False

*Proof or Counterexample:*

C3. (5 points) Let $A$ and $B$ be $n \times n$ matrices. Suppose that $T_A$ is surjective and that $B = E_1 \cdots E_r$ is a product of elementary matrices. Let $F : \mathbb{R}^n \to \mathbb{R}^n$ be the linear transformation $F = T_A \circ T_B$. Show that $\ker(F) = \{\vec{0}\}$.

*Proof.*

**YOU MUST SUBMIT THIS PAGE.**

If you would like work on this page scored, then clearly indicate to which question the work belongs and indicate on the page containing the original question that there is work on this page to score. **Do not remove this page** from your test packet.

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