CHAPTER 7 ACTIVITY PACKET NAME:

Instructions. This packet is due on Quercus no later than **11:59pm on Monday, March 2nd**. Please complete your work directly on this packet. We will spend time together during lecture working on most or all of the activities in this packet. You are responsible for completing all portions of this packet, including lecture activities not discussed in class, and completing the definitions included in the packet. Solutions will be posted to the course website after the assignment due date.

Definition 7.1. The UNIT SQUARE is the subset of $\mathbb{R}^2$ given by ...

Lecture Activity 7.1. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$F(\vec{e}_1) = 3\vec{e}_1 \text{ and } F(\vec{e}_2) = 2\vec{e}_2.$$

P1. Sketch a picture of the unit square S.

P2. Consider the set $F(S) := \{F(\vec{v}) : \vec{v} \in S\}$. Use the fact that F is linear to show that

$$F(S) = \{\alpha_1(3\vec{e}_1) + \alpha_2(2\vec{e}_2) : 0 \leq \alpha_1, \alpha_2 \leq 1\}.$$

P3. Use your work in P2 to sketch a picture of $F(S)$ as a subset of $\mathbb{R}^2$.

P4. Sketch the image of the "standard coordinate grid" for $\mathbb{R}^2$ under F.

Lecture Activity 7.2. Let $G : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$G(\vec{e}_1) = \vec{e}_1 + \vec{e}_2 \text{ and } G(\vec{e}_2) = 2\vec{e}_2.$$

P1. Consider the set $G(S) := \{G(\vec{v}) : \vec{v} \in S\}$. Use the fact that G is linear to show that

$$G(S) = \{\alpha_1(\vec{e}_1 + \vec{e}_2) + \alpha_2(2\vec{e}_2) : 0 \leq \alpha_1, \alpha_2 \leq 1\}.$$

P2. Use your work in P1 to sketch a picture of $G(S)$ as a subset of $\mathbb{R}^2$. Then, sketch the image of the "standard coordinate grid" for $\mathbb{R}^2$ under G.

Definition 7.3. An ordered basis $\{\vec{b}_1, \vec{b}_2\}$ for $\mathbb{R}^2$ is called POSITIVELY ORIENTED if ...

Otherwise, the basis is called NEGATIVELY ORIENTED.

Lecture Activity 7.3. Find the orientation for the following ordered bases for $\mathbb{R}^2$.

P1. $\mathcal{B} = \{\vec{b}_1, \vec{b}_2\}$ where

$$\vec{b}_1 = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \text{ and } \vec{b}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

P2. $\mathcal{C} = \{\vec{c}_1, \vec{c}_2\}$ where

$$\vec{c}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } \vec{c}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Definition 7.4. Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Then, the DETERMINANT of F, denoted by $\det(F)$, is ...

If A is a 2×2 matrix, the DETERMINANT OF A, denoted by $\det(A)$, is ...

Lecture Activity 7.4. Find the determinant of matrices

$$A = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

Lecture Activity 7.5. In this activity, we develop a method to calculate the determinant of a 2×2 matrix completely algebraically. Let

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

For simplicity we'll assume that the set $\left\{ \begin{pmatrix} a \\ c \end{pmatrix}, \begin{pmatrix} b \\ d \end{pmatrix} \right\}$ is a positively oriented ordered basis with $d \neq 0$ and both vectors are in the first quadrant.

P1. Recall that the area of a parallelogram can be computed as the product of its base times its height. Use this observation to calculate the determinant of

$$\begin{pmatrix} e & b \\ 0 & d \end{pmatrix}.$$

P2. Show that $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \det \begin{pmatrix} a - \frac{bc}{d} & b \\ 0 & d \end{pmatrix}$.

P3. Use the previous two parts to conclude that

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc.$$

Lecture Activity 7.6. Calculate the determinant of the following functions. Given your calculation, what can be said about how the function defined by the given matrix transforms the domain space $\mathbb{R}^2$?

P1. $A = \begin{pmatrix} 2 & 4 \\ 1 & 5/2 \end{pmatrix}$

P2. $B = \begin{pmatrix} 1/2 & 5 \\ 4 & 2 \end{pmatrix}$

P3. $C = \begin{pmatrix} 4 & 1/2 \\ 8 & 1 \end{pmatrix}$

P4. $D = \begin{pmatrix} 3 & 1/2 \\ 2 & 1/2 \end{pmatrix}$

Definition 7.6. The UNIT CUBE is the subset of $\mathbb{R}^3$ given by ...

Lecture Activity 7.7. Find the orientation of the following ordered bases for $\mathbb{R}^3$.

P1. $\mathcal{B} = \{\vec{b}_1, \vec{b}_2, \vec{b}_3\}$ where

$$\vec{b}_1 = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \text{ and } \vec{b}_3 = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}.$$

P2. $\mathcal{C} = \{\vec{c}_1, \vec{c}_2, \vec{c}_3\}$ where

$$\vec{c}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \vec{c}_2 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, \vec{c}_3 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}.$$

Lecture Activity 7.8. Calculate the determinant of the matrices

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 1 \end{pmatrix}, \text{ and } B = \begin{pmatrix} 0 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & -1 & 0 \end{pmatrix}$$

Lecture Activity 7.9. Use Proposition 7.10 to calculate $\det(A)$, where

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 3 & 1 \\ 1 & -1 & -1 \end{pmatrix}.$$

Given this calculation, what can you say about the matrix transformation $T_A : \mathbb{R}^3 \to \mathbb{R}^3$?

Definition 7.12. For an $n \times n$ matrix $A = \left(a_{ij}\right)$, the ij-MINOR of A is ...

Definition 7.13. Let A be the $n \times n$ matrix with ij-entry equal to a_{ij}. We define the determinant of A by the following *cofactor expansion* formula:

Lecture Activity 7.10. Find the determinant of the following matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 2 \end{pmatrix}.$$

Proposition 7.15 (Determinant Properties)**.** Let A be an $n \times n$ matrix.

1. If $B = A^\top$ is the transpose of A, then $\det(B) = \boxed{}$

2. If B is obtained by interchanging two rows of A, then $\det(B) = \boxed{}$

3. If B is obtained by multiplying one row of A by a constant c, then $\det(B) = \boxed{}$

4. If B is obtained by replacing a row of A by that row and a scalar multiple of another row of A, then $\det(B) = \boxed{}$

5. If B is any $n \times n$ matrix, then $\det(AB) = \boxed{}$

Lecture Activity 7.11. Use Proposition 7.15 to calculate the determinant of the following matrix

$$A = \begin{pmatrix} 1 & 0 & 3 & 0 \\ 1 & 0 & 2 & 0 \\ 4 & 0 & 0 & 1 \\ 1 & 2 & 3 & 0 \end{pmatrix}$$