Instructions. This packet is due on Quercus no later than **11:59pm on Monday, November 3rd**. Please complete your work directly on this packet. We will spend time together during lecture working on most or all of the activities in this packet. You are responsible for completing all portions of this packet, including lecture activities not discussed in class, and completing the definitions included in the packet. Solutions will be posted to the course website after the assignment due date.

Definition 8.1. Let A be an $n \times n$ matrix. A non-zero vector $\vec{v}$ is an EIGENVECTOR of A if ...

The scalar λ is called an EIGENVALUE of A.

Lecture Activity 8.1. For each of the following matrix-vector pairs, determine whether $\vec{v}$ is an eigenvector of the matrix A. If it is, find the corresponding eigenvalue λ.

P1. $A = \begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

P2. $B = \begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

P3. $C = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

Proposition 8.2. For an $n \times n$ matrix A, the set of eigenvectors of A corresponding to an eigenvalue λ is equal to the nonzero vectors in $\mathrm{Nul}(A - \lambda I_n)$.

Prove Proposition 8.2.

Definition 8.3. Let A be an $n \times n$ matrix with eigenvalue λ.

1. The λ-Eigenspace of A is the vector subspace of $\mathbb{R}^n$ defined by ...

2. The GEOMETRIC MULTIPLICITY of λ is ...

Lecture Activity 8.2. Let $A = \begin{pmatrix} 3 & 2 \\ 3 & 8 \end{pmatrix}$, and recall from Lecture Activity 8.1 that $\lambda = 2$ is an eigenvalue of A

P1. Find the 2-Eigenspace of A.

P2. Find the geometric multiplicity of $\lambda = 2$.

Proposition 8.5. A real number λ is an eigenvalue of an $n \times n$ matrix A if and only if

$$\det(A - \lambda I_n) = 0.$$

Prove Proposition 8.5.

Definition 8.6. For an $n \times n$ matrix A, the CHARACTERISTIC POLYNOMIAL of A is ...

Lecture Activity 8.3. Find the characteristic polynomial of the following matrices. Then, use Proposition 8.5 to find the eigenvalues of each matrix.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$