Instructions. This packet is due on Quercus no later than **11:59pm on Tuesday, October 14th**. Please complete your work directly on this packet. We will spend time together during lecture working on most or all of the activities in this packet. You are responsible for completing all portions of this packet, including lecture activities not discussed in class, and completing the definitions included in the packet. Solutions will be posted to the course website after the assignment due date.

Lecture Activity 6.1. Let $F : \mathbb{R}^n \to \mathbb{R}^m$ and $G : \mathbb{R}^n \to \mathbb{R}^m$ be linear transformations.

P1. Show that $F + G$ is a linear transformations.

P2. Let $F = T_A$ and $G = T_B$ where

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \text{ and } B = \begin{pmatrix} -1 & 3 \\ 1 & 2 \end{pmatrix}.$$

By P1, we know that $F + G$ is linear. Find the defining matrix M_{F+G}.

Definition 6.1. Let $A = \begin{pmatrix} \vec{v}_1 & \cdots & \vec{v}_n \end{pmatrix}$ and $B = \begin{pmatrix} \vec{w}_1 & \cdots & \vec{w}_n \end{pmatrix}$ be $m \times n$ matrices and $c \in \mathbb{R}$ be a scalar.

1. The SUM of A and B is the $m \times n$ matrix given by ...

2. The SCALAR PRODUCT of A with c is the $m \times n$ matrix given by ...

Lecture Activity 6.2. Let $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$. Find the matrix A given that

$$2(A + (B + 3A)) = 7A - (B + A).$$

Lecture Activity 6.3. Let $F : \mathbb{R}^k \to \mathbb{R}^m$ and $G : \mathbb{R}^n \to \mathbb{R}^k$ be linear transformations with defining matrices

$$A = M_F \text{ and } B = M_G.$$

Recall from Chapter Exercise 4.3 that the composition of linear functions is linear. Show that the defining matrix $M = M_{F \circ G}$ of the composition $F \circ G$ is given by

$$M_{F\circ G} = \begin{pmatrix} A\vec{b}_1 & A\vec{b}_2 & \cdots & A\vec{b}_n \end{pmatrix}$$

where $B = \begin{pmatrix} \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_n \end{pmatrix}$.

Definition 6.3. Let A be an $m \times k$ matrix and $B = \begin{pmatrix} \vec{b}_1 & \cdots & \vec{b}_n \end{pmatrix}$ be a $k \times n$ matrix. Then, the MATRIX PRODUCT of A and B is the $m \times n$ matrix ...

As we saw in Lecture Activity 6.3, note that $T_{AB} = T_A \circ T_B$.

Lecture Activity 6.4. Let

$$A = \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 2 & 3 & 1 \end{pmatrix}, \ B = \begin{pmatrix} 0 & -1 \\ 3 & 0 \\ 1 & 0 \\ -1 & 4 \end{pmatrix} \text{ and } C = \begin{pmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & -2 & 1 \end{pmatrix}.$$

Calculate all possible matrix products AB, BA, AC, CA, BC, CB. If a matrix product is not defined, explain why not.

Lecture Activity 6.5. Let $F : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, and suppose that the inverse function $F^{-1} : \mathbb{R}^m \to \mathbb{R}^n$ is known to exist.

P1. Use Lecture Activity 4.9 to show that $m = n$.

P2. Show that F^{-1} is a linear transformation.

P3. Suppose that F has defining matrix $M_F = A$ and F^{-1} has defining matrix $M_{F^{-1}} = B$. What matrix does AB need to be equal to? What about BA?

Definition 6.6. The IDENTITY MATRIX I_n is ...

Definition 6.7. Let A be an $n \times n$ matrix. Then INVERSE OF A, if it exists, is ...

Lecture Activity 6.6. Use Theorem 6.11 to find the inverse of $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{pmatrix}$.

Definition 6.12. An $n \times n$ matrix is called ELEMENTARY if ...

Lecture Activity 6.7. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

P1. Let E_1 be the elementary matrix obtained by performing the row operation $R_1 \leftrightarrow R_3$ to I_3. Find E_1, and then calculate $E_1 A$. What do you notice?

P2. Let E_2 be the elementary matrix obtained by performing the row operation $5R_2$ to I_3. Find E_2 and then calculate $E_2 A$. What do you notice?

P3. Let E_3 be the elementary matrix obtained by performing the row operation $R_1 + 2R_2$ to I_3. Find E_3 and then calculate $E_3 A$. What do you notice?

Lecture Activity 6.8. Let's see how we can capture Gauss-Jordan via matrix products. Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & -1 \end{pmatrix}$$

from Lecture Activity 6.6.

P1. Use your work in Lecture Activity 6.6 to find elementary matrices $E_1, \ldots, E_k$ so that

$$E_1 \cdots E_k A = I_3.$$

P2. Reverse your work from Lecture Activity 6.7 to show that $I_3 \sim A$. That is, find a sequence of elementary operations to perform to I_3 to obtain A.

P3. Find elementary matrices $\tilde{E}_1, \dots, \tilde{E}_k$ so that $\tilde{E}_k \cdots \tilde{E}_1 I_3 = A$. Conclude that A is a product of elementary matrices.