CHAPTER 1 ACTIVITY PACKET NAME:

Instructions. This packet is due on Quercus no later than **11:59pm on Monday, September 8th**. Please complete your work directly on this packet. We will spend time together during lecture working on most or all of the examples and lecture activities in this packet. You are responsible for completing all portions of this packet, including lecture activities not discussed in class, and completing the definitions included in the packet. Solutions will be posted to the course website after the assignment due date.

Note: Typos/mistakes found after printing are noted in red text.

Lecture Activity 1.1. Find a system of linear equations that models each of the problems below.

P1. *Blending Problems* arise in contexts where several inputs are combined to produce a final product with specific desired properties. Given the characteristics of each input, the goal is to determine how much of each input is needed to meet the target requirements.

Suppose you are planning a meal using three foods. Each unit of food contains a different combination of protein, carbohydrates, and fat, as shown below:

Food	Protein (g)	Carbs (g)	Fat (g)
Food A	2	3	1
Food B	1	1	2
Food C	4	2	3

If you want your meal to contain exactly 30 grams of protein, 25 grams of carbs, and 20 grams of fat, how many units of each food should you include in your meal?

P2. Given a collection of data, it is often useful to find a mathematical model that fits the data exactly. This process, called *interpolation*, allows us to estimate missing values and understand the underlying pattern of our data.

Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ that passes through all of the following points: $(0, -2), (1, 1), (2, 6)$, and $(3, 10)$.

P3. *Flow conservation* problems arise in systems where something moves through a network so that at each junction, the total amount flowing in must equal the total amount flowing out.

The University of Toronto supplies water to campus buildings from the city system through a one-way distribution network. In this system, the flow into each building must equal the flow out, unless some water is used on-site. Three connected buildings are involved: ROB (Robarts Library), SS (Sidney Smith Hall), and BA (Bahen Centre). Water is delivered between the buildings as follows:

- The city sends 120 liters of water per minute to ROB,
- ROB uses 35 liters of water per minute and sends the rest to SS,
- SS uses 40 liters of water per minute and sends the rest to BA, and
- BA uses 45 liters of water per minute and sends the rest to ROB.

Describe the rate at which water needs to flow between each of the buildings in order for the system to be conserved (that is, no water is lost or stored)?

Lecture Activity 1.2. Find all solutions to the the system of linear equations that have the following augmented matrices

P1. $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 2 & 2 \end{array}\right)$

P2. $\left(\begin{array}{ccc|c} 0 & 2 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right)$

P3. $\left(\begin{array}{ccc|c} 2 & 2 & 1 & 2 \\ 0 & 0 & 3 & 6 \end{array}\right)$

Definition 1.13. The PIVOT of a row in a matrix is . . .

Definition 1.15. A matrix is in ROW ECHELON FORM if . . .

Definition 1.16. A matrix is in REDUCED ROW ECHELON FORM if . . .

Lecture Activity 1.3. Determine which of the following matrices are in row echelon form, reduced row echelon form, or neither.

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 0 & 1 \\ 3 & 4 & -1/2 \end{pmatrix}, \; B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \; C = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 2 & 1 \end{pmatrix},$$

$$D = \begin{pmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \; F = \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Lecture Activity 1.4. Give an example of matrix A that satisfies all of the following conditions:

- A is the augmented matrix for a system of 4 linear equations in 4 variables.
- A is in reduced row echelon form.
- A has exactly three pivots.
- A contains at least one entry other than 0 or 1.

Lecture Activity 1.5. Find $\mathrm{rref}(A)$, where

$$A = \begin{pmatrix} 1 & 1 & 1 & 7 & -1 \\ 0 & 1 & 1 & 4 & -1 \\ -1 & 2 & 2 & 5 & -2 \end{pmatrix}.$$

Definition 1.23. A system of linear equations is called CONSISTENT if it has at least one solution. If the system has no solutions, it is called INCONSISTENT.

Definition 1.24. Let C be the coefficient matrix for a consistent system of linear equations in variables $x_1, \ldots, x_n$.

1. We say that x_i is a BASIC VARIABLE for the system if ...

2. We say that x_i is a FREE VARIABLE for the system if ...

Lecture Activity 1.6. Let A be the matrix from Lecture Activity 1.5, and consider the system of linear equations in variables x, y, z, w which has A as its augmented matrix.

P1. Determine the free variables and basic variables of the system.

P2. Give a parametric description of the solution set to this system.

Theorem 1.27. (Rouché-Capelli). Suppose that a system of linear equation has augmented matrix A and coefficient matrix C. Then,

1. The system is inconsistent if and only if ...

2. The system has exactly one solution if and only if ...

3. The system has infinitely many solutions if and only if ...

Lecture Activity 1.7. Suppose that we're given a set of n data points that lie exactly on the graph of some unknown cubic function. What's the minimum number of points needed to guarantee there's *exactly one* cubic function passing through them? Explain your answer.