$4Tu$ AND THE "TRUE" SKEIN KHOVANOV HOMOLOGY

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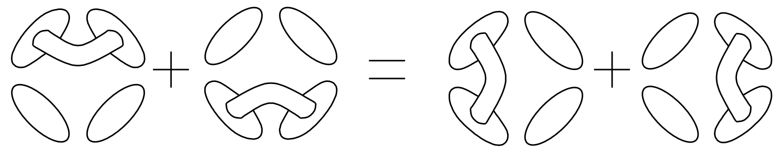
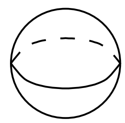
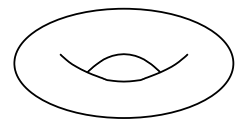
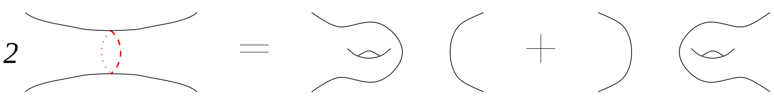
$4Tu$: 

S :  T : 

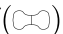
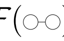
What is it good for?

(1) Cutting necks:



(2) Recovers the good old Khovanov theory,

$\mathcal{F}($$) = \epsilon : \begin{cases} 1 \mapsto v_+ \end{cases}$ $\mathcal{F}($$) = \eta : \begin{cases} v_+ \mapsto 0 \\ v_- \mapsto 1 \end{cases}$

$\mathcal{F}($$) = \Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases}$ $\mathcal{F}($$) = m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0. \end{cases}$

(3) Trivially extends to tangles.
(4) Well suited to prove invariance for cobordisms.
(5) Recovers Lee's theory,

$$\Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \mapsto v_- \otimes v_- + v_+ \otimes v_+ \end{cases} \qquad m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto v_+. \end{cases}$$

(6) Leads to a new theory (over $\mathbb{Z}/2$ and with $\deg h = -2$),

$$\Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ + hv_+ \otimes v_+ \\ v_- \mapsto v_- \otimes v_- \end{cases} \qquad m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto hv_-. \end{cases}$$

(7) Trivially extends to knots on surfaces.
(8) Non-trivially recovers Khovanov's c,

$$\epsilon : \begin{cases} 1 \mapsto v_+ \end{cases} \qquad \eta : \begin{cases} v_+ \mapsto 0 \\ v_- \mapsto -c \end{cases}$$

$$\Delta : \begin{cases} v_+ \mapsto v_+ \otimes v_- + v_- \otimes v_+ + cv_- \otimes v_- \\ v_- \mapsto v_- \otimes v_- \end{cases} \qquad m : \begin{cases} v_+ \otimes v_- \mapsto v_- & v_+ \otimes v_+ \mapsto v_+ \\ v_- \otimes v_+ \mapsto v_- & v_- \otimes v_- \mapsto 0. \end{cases}$$

(Added June 29, 2004: what appeared to work didn't quite. The recovery of Khovanov's c remains open).

"God created the knots, all else in topology is the work of man."

Leopold Kronecker (modified)

URL: `http://www.math.toronto.edu/~drorbn/papers/Cobordism (and see the ''GWU'' handout)`

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