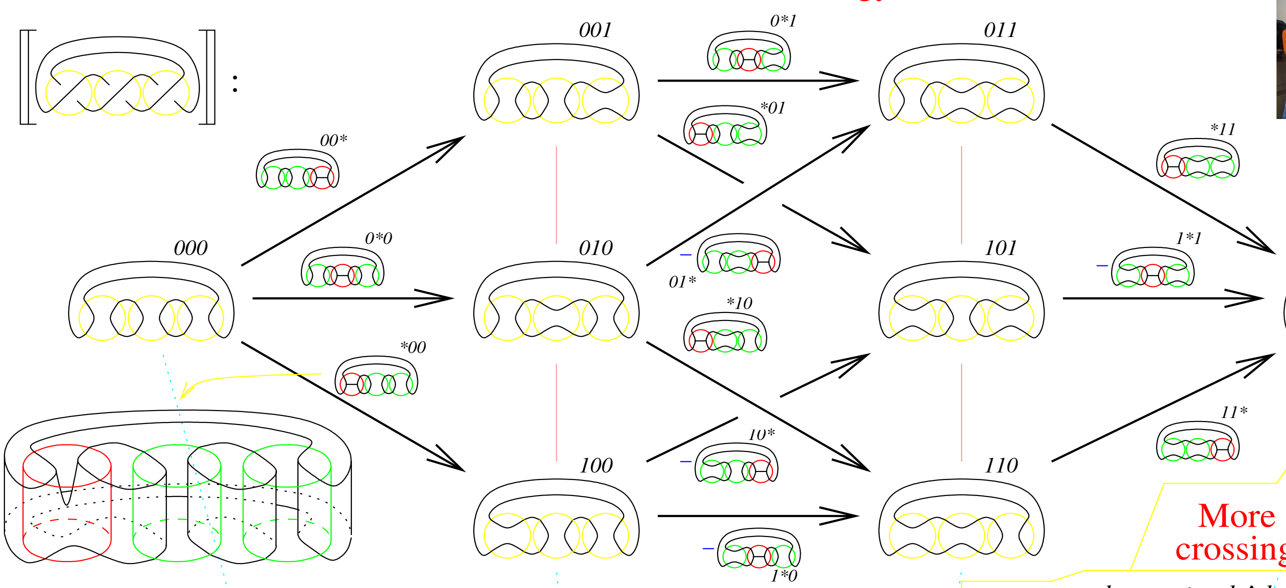
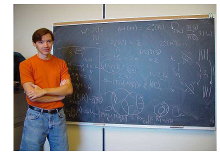
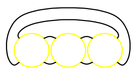
Khovanov Homology

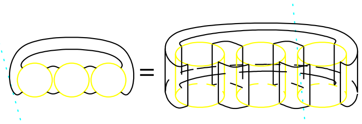



Mikhail Khovanov

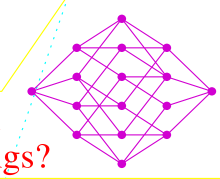
What is it?

A cube for each knot/link projection;

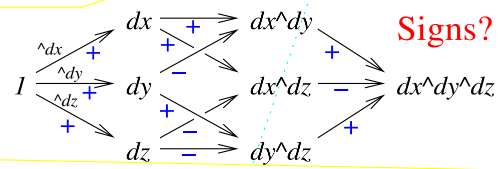
Vertices: All fillings of  with) (or with ⌣⌢ .

Edges: All fillings of $I\times$  with $I\times$) (= (green cobordism) or with $I\times$ ⌣⌢ = (green cobordism) and precisely one (red saddle cobordism) .

More crossings?



Signs?



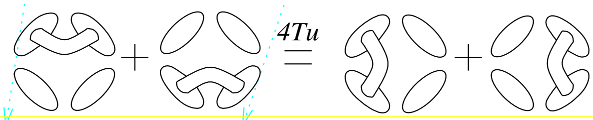
Where does it live?

In *Kom(Mat(<Cob> / {S, T, 4Tu})) / homotopy* :

Kom: Complexes *Cob: Cobordisms*

<...>: Formal lin. comb. *Mat: Matrices*

S: (sphere) =0 *T:* (torus) =2



Jones/Kauffman?

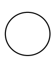
A TQFT takes it to a complex whose graded Euler characteristic is the Jones polynomial.

$$V^{\otimes 3} \longrightarrow (V^{\otimes 2}\oplus V^{\otimes 2}\oplus V^{\otimes 2})\{1\} \longrightarrow (V\oplus V\oplus V)\{2\} \longrightarrow V^{\otimes 2}\{3\}$$

$$(q+q^{-1})^3 \qquad 3q(q+q^{-1})^2 \qquad 3q^2(q+q^{-1}) \qquad q^3(q+q^{-1})^2$$


V. Jones

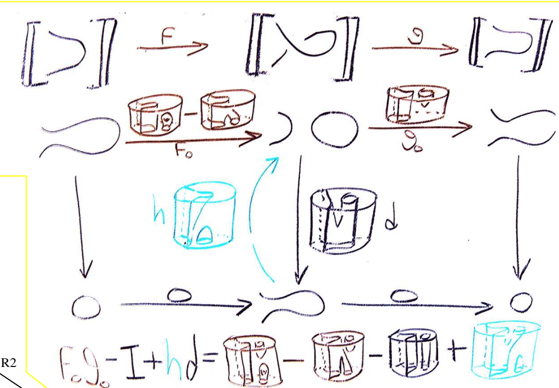

L. Kauffman

The key point:  $\longrightarrow$ $V=\langle v_+, v_-\rangle,\quad \deg v_\pm = \pm 1$

$q\text{-dim}V = q+q^{-1}$

But is it invariant?

(With similar proofs for R–II and R–III)

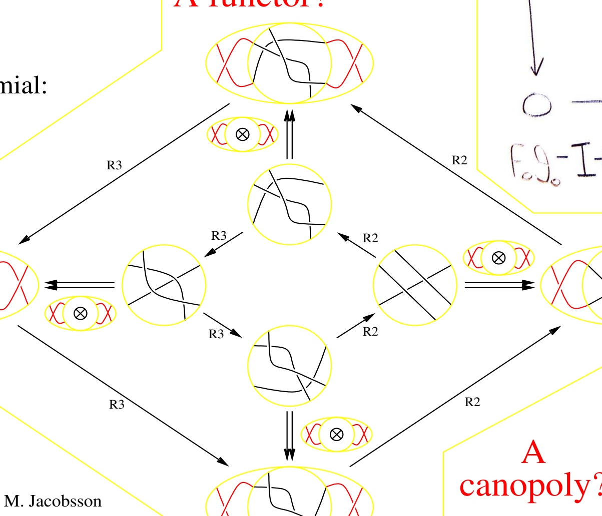


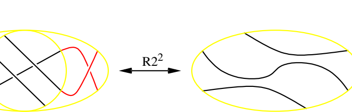
Why is it interesting?

1. It is stronger than the Jones polynomial.
2. It is less understood than the Jones polynomial:
   - a. Does it have a topological interpretation?
   - b. Does it have a "physical" interpretation?
   - c. Does it also work for other quantum invariants?
   - d. Does it work for manifolds and for knots in manifolds?
   - e. Is there a relation with finite–type invariants?
   - f. Does it work for "virtual knots"?
3. Jacobsson, Khovanov: It is a functor!!! (from knots and cobordisms to complexes and morphisms)


M. Jacobsson

A functor?





A canopoly?

See http://www.math.toronto.edu/~drorbn/papers/Cobordism

Dror Bar–Natan, Warszawa, July 2003.

More at http://www.math.toronto.edu/~drorbn/Talks/UWO-040213/