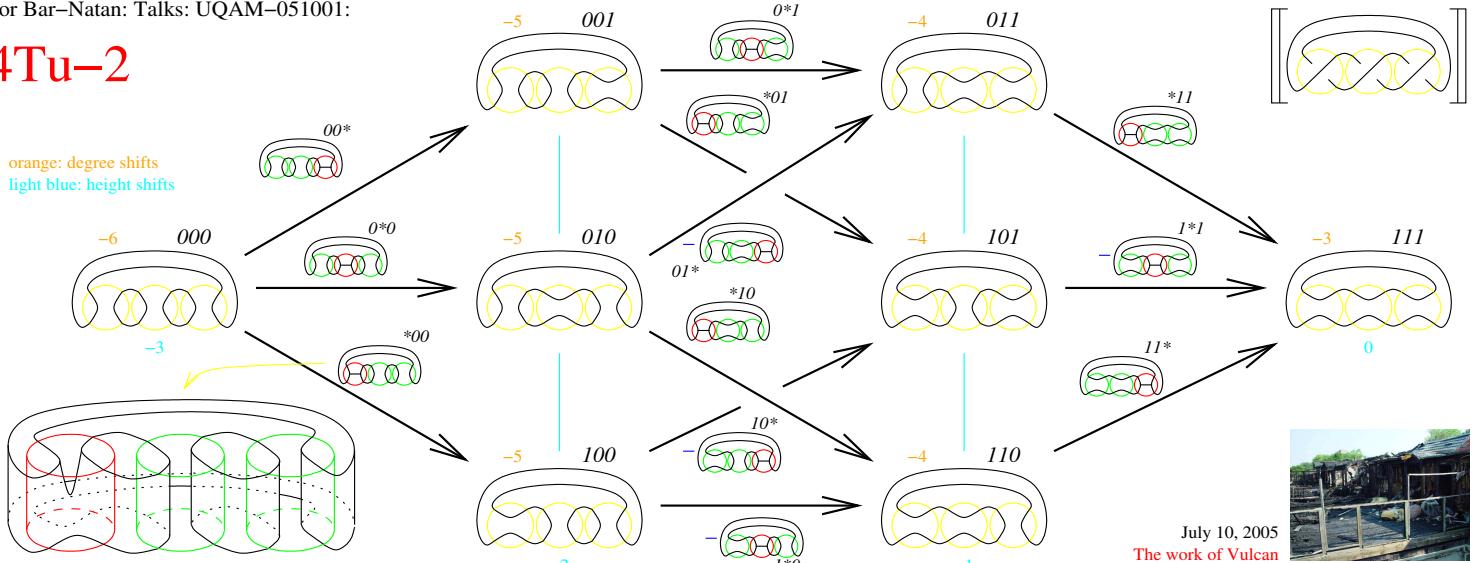


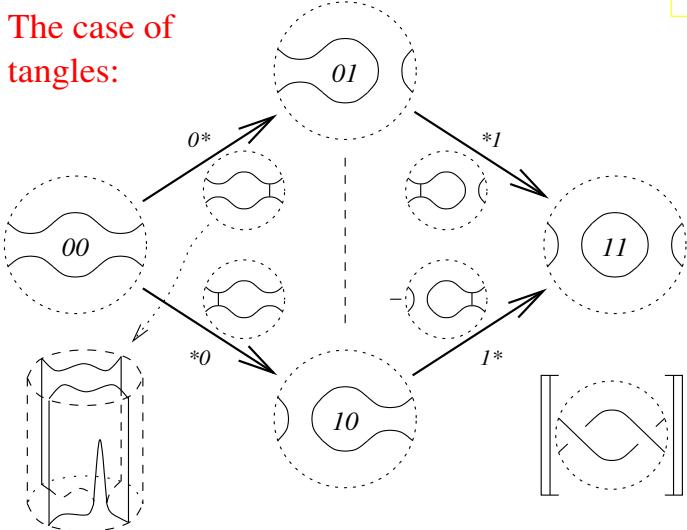
## 4Tu-2



## General Crossings

$$\begin{array}{c} \text{Diagram 1: } \text{Crossing} \rightarrow \left( \begin{array}{c} \text{Crossing} \\ 0 \end{array} \right) \xrightarrow{\text{Crossing}} \left( \begin{array}{c} \text{Crossing} \\ +1 \end{array} \right) \\ \text{Diagram 2: } \text{Crossing} \rightarrow \left( \begin{array}{c} \text{Crossing} \\ -1 \end{array} \right) \xrightarrow{\text{Crossing}} \left( \begin{array}{c} \text{Crossing} \\ -2 \end{array} \right) \end{array}$$

## The case of tangles:



## The work of Naot.

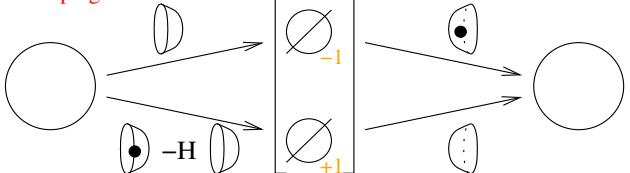
&lt;surfaces&gt;/4Tu is freely generated by Shrek surfaces



A Shrek surface with 7 boundaries (one distinguished), 3 handles and 2 tubes

Let  $\bullet$  denote a tube to the distinguished component (the curtain), and let  $H$  denote a handle on the curtain. Then

Delooping:

... so the invariant is valued in complexes over a category with just one object and morphisms in  $Z[H]$ ; all is graded and  $\deg H = -2$ .

Where does it live? In *Kom(Mat(<Cob> / {S, T, 4Tu})) / homotopy*  
*Kom*: Complexes *Mat*: Matrices *Cob*: Cobordisms  $<\dots>$ : Formal lin. comb.

$$\begin{array}{c} S: \text{Diagram} = 0 \\ T: \text{Diagram} = 2 \end{array}$$

$$\text{Diagram} + \text{Diagram} \stackrel{4Tu}{=} \text{Diagram} + \text{Diagram}$$

## Invariant!



$$\begin{array}{c} \text{Diagram} \xrightarrow{F} \text{Diagram} \xrightarrow{G} \text{Diagram} \\ \text{Diagram} \xrightarrow{F_0} \text{Diagram} \xrightarrow{G_0} \text{Diagram} \\ \text{Diagram} \xrightarrow{h} \text{Diagram} \xrightarrow{G} \text{Diagram} \\ \text{Diagram} - \text{Diagram} + \text{Diagram} = \text{Diagram} - \text{Diagram} - \text{Diagram} + \text{Diagram} \end{array}$$

The Reduction Lemma. If  $\phi$  is an isomorphism then the complex

$$[C] \xrightarrow{\begin{pmatrix} \alpha \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & \delta \\ \gamma & \epsilon \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix} \mu & \nu \end{pmatrix}} [F]$$

is isomorphic to the (direct sum) complex

$$[C] \xrightarrow{\begin{pmatrix} 0 \\ \beta \end{pmatrix}} \begin{bmatrix} b_1 \\ D \end{bmatrix} \xrightarrow{\begin{pmatrix} \phi & 0 \\ 0 & \epsilon - \gamma\phi^{-1}\delta \end{pmatrix}} \begin{bmatrix} b_2 \\ E \end{bmatrix} \xrightarrow{\begin{pmatrix} 0 & \nu \end{pmatrix}} [F]$$

## The work of Green.



The universal invariant of the left-handed trefoil is

$$\begin{array}{c} \text{Diagram} \xrightarrow{H} \text{Diagram} \xrightarrow{G} 0 \xrightarrow{F} \text{Diagram} \\ \text{Diagram} \xrightarrow{-8} \text{Diagram} \xrightarrow{-6} \text{Diagram} \xrightarrow{-1} \text{Diagram} \xrightarrow{0} \text{Diagram} \xrightarrow{-2} \text{Diagram} \end{array}$$

standard data:  $\begin{array}{c} -1 \\ -3 \\ -5 \\ -7 \\ -9 \\ -3 \\ -2 \\ -1 \end{array}$

## Some functors.

$$\begin{array}{c} \text{classical} \quad \text{reduced} \quad \text{Lee} \quad ? \\ H \mapsto \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array} \quad \begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array} \quad \begin{array}{c} \frac{\mathbb{Z}[X]}{X^2 - hX - t} \\ \begin{array}{c} i \\ + \\ - \\ + \\ - \\ h \\ 2 \\ 2 \end{array} \end{array} \\ \begin{array}{c} \langle + \rangle \\ \langle - \rangle \end{array} \quad \begin{array}{c} \langle 0 \rangle \\ \langle 0 \rangle \end{array} \quad \begin{array}{c} \langle 2 \rangle \\ \langle 0 \rangle \\ \langle 0 \rangle \\ \langle -2 \rangle \\ \langle -2 \rangle \end{array} \end{array}$$

(Lee's spectral sequence and Rasmussen's invariant also recoverable)

<http://www.math.toronto.edu/~drorbn/Talks/UQAM-051001/>