

We Mean Business

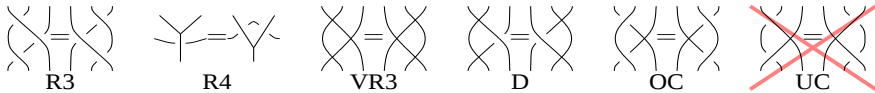
Trivalent (framed) w-tangles:

further operations: delete, unzip.

$$wTT = CA \langle \text{diagram} \rangle / R123, R4 \text{ (for vertices)}, F, OC.$$

$$= PA \langle \text{diagram} \rangle / R1234, F, VR1234, D, OC.$$

(=tangles in thick surfaces, modulo stabilization)



Partial Dictionary.

$$(R, F) \leftrightarrow (\text{diagram}, \text{diagram}) \quad (r, t) \leftrightarrow (\text{diagram}, \text{diagram})$$

$$R^{12}R^{13}R^{23} = R^{23}R^{13}R^{12} \leftrightarrow \text{diagram} = \text{diagram}$$

$$FF^! = I \leftrightarrow \text{diagram} \xrightarrow{\text{unzip}} \text{diagram}$$

$$F^{-1}e(x+y)F = e(x)e(y)$$

$$F^{23}R^{123} = R^{12}R^{13}F^{23} \leftrightarrow \text{diagram} = \text{diagram}$$

$$R^{12,3} = R^{13}R^{23}$$

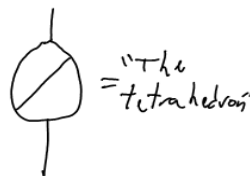
$$F^{123}R^{12,3} = R^{13}R^{23}F^{12,3} \leftrightarrow \text{diagram} = \text{diagram}$$

(unforbidding FI makes this automatic)

$$RF^{21}e(-t) = F \leftrightarrow \text{diagram} = \text{diagram}$$

$$\Phi = (F^{12,3})^{-1}(F^{12})^{-1}F^{2,3}F^{1,23} \leftrightarrow$$

$$\Phi \in \text{sdw} \leftrightarrow \text{diagram} = \text{diagram}$$



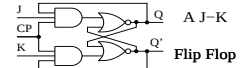
The pentagon and The hexagons Follow, with a minor twist, from the fact that we have an unzip behaved invariant of KTG's.

Circuit Algebras

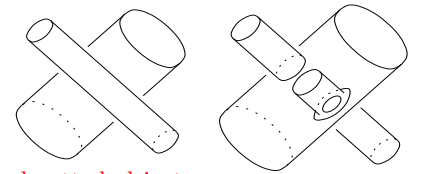
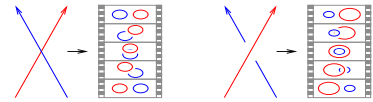
* Have "circuits" with "ends"

* Can be wired arbitrarily.

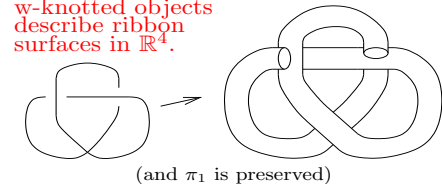
* May have "relations" – de-Morgan, etc.



w-braids describe flying rings:



w-knotted objects describe ribbon surfaces in $\mathbb{R}^4$.



For the Experienced (and sharp-eyed)

The "Chord Diagrams" — $\mathcal{A}_n^{wt}$.

As we did for quandles, substitute $\text{diagram} \mapsto \text{diagram} + (\text{diagram} - \text{diagram}) = \text{diagram} + \text{diagram}$ into the various moves, to get relations. Also switch to arrow diagram language: $\text{diagram} \leftrightarrow \text{diagram}$. Get: $\text{diagram} = \text{diagram} \mapsto \text{diagram} = \text{diagram}$ (tails commute) $R3 \mapsto \text{diagram} - \text{diagram} = \text{diagram} - \text{diagram}$ (really 4T) $R4 \mapsto \text{diagram} + \text{diagram} = 0$ (vertex invariance)

The "Jacobi Diagrams" — $\mathcal{A}_n^{cc}$.

Theorem. $\mathcal{A}_n^{wt}$ is $\mathcal{A}_n^{cc}$ is $\mathcal{U}(\text{tder}_n)$.

Here $\mathcal{A}_n^{cc}$ is $\{ \text{diagram} \} / \text{rules}$ trivalent directed trees with only 2-in 1-out vertices. In tensorland, this is "Co-commutative Lie-bialgebras". Rules: tails commute $\text{diagram} = \text{diagram}$ Heads satisfy the only possible STU: $\text{diagram} - \text{diagram} = \text{diagram} - \text{diagram}$ + also IHX and vertex invariance

The Map $\alpha: \mathcal{A}_n^{tree} \rightarrow \mathcal{A}_n^{cc}$:

Theorem. α is an injection on $\mathcal{A}_n^{tree} \cong \mathcal{U}(\text{sdern})$. Furthermore, there is a simple characterization of $\text{im } \alpha$, so we can tell "an arrowless element" when we see it.

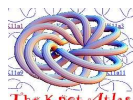
The Main Theorem. (approximate, false as stated) F 's in Sol_0^T are in a bijective correspondence with tree-level associators for ordinary parenthesized tangles (or ordinary knotted trivalent graphs) / with homomorphic expansions for trivalent w-tangles / with solutions of the Kashiwara-Vergne problem.

Extra. Restricted to knots, we get precisely the Alexander polynomial.

Disclaimer. Orientations, rotation numbers, framings, the vertical direction and the cyclic symmetry of the vertex may still make everything uglier. I hope not.

"God created the knots, all else in topology is the work of mortals"

Leopold Kronecker (paraphrased)



Visit!

Edit!

<http://katlas.org>

This handout and further links are at <http://www.math.toronto.edu/~drorbn/Talks/MSRI-0808/>