Overview of Khovanov Homology (2)

Cob:

Mat(C):

Complexes:

$$\Omega = \left(\Omega^{-n_-} \longrightarrow \Omega^{-n_-+1} \longrightarrow \cdots \longrightarrow \Omega^{n_+} \right)$$

Morphisms:

Homotopies:

$$F^r - G^r = h^{r+1}d^r + d^{r-1}h^r$$

All arrows in an arbitrary additive category!

Invariance under R2.

After delooping:

The Jones polynomial:

Definition. $\hat{J} : \asymp \mapsto q)(- q^2 \smile$, $\quad \hat{J} : \asymp \mapsto -q^{-2}\smile + q^{-1})($,

valued in xing−free tangles mod

$$\bigcirc = q + q^{-1}$$

Invariance under R2:
$$\hat{J} : \gtrless \mapsto -q^{-1}\smile + \gtrless + \smile - q\smile$$
$$= -q^{-1}\smile +)(+ (q + q^{-1})\smile - q\smile$$
$$=)(\qquad \square$$

Kh(T(7,6)).

In 1 day [laptop image] says

$\dim_j H_r$ is given by:

j \ r	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
57																			1	1
55																	1	1		
53														1	2	1	1			
51											1	1		2	1					
49												3	1		1					
47											3	1								
45									2		1	2								
43							1		1	2										
41						1		1	2	1										
39						1	1		1											
37				1	1		1													
35					1															
33			1																	
31	1																			
29	1																			

the alternating knots zone

Functoriality / cobordisms.

M. Jacobsson
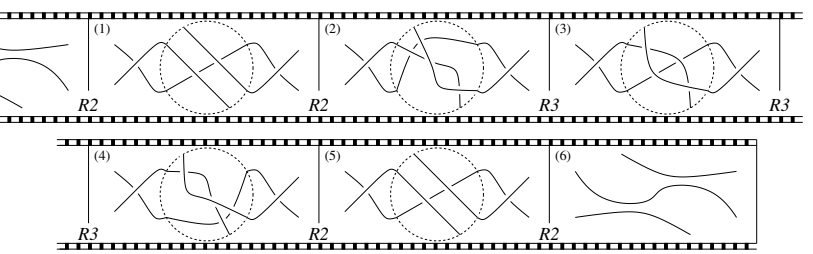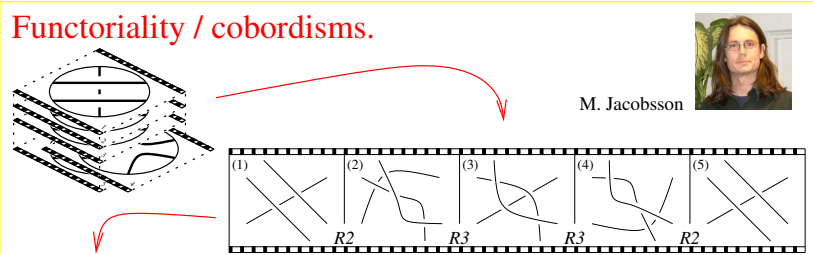
J. Rasmussen: This leads to a no−analysis proof of Milnor's conjecture

Conjecture: (I. Frenkel, though he may disown this version)

1. Every object in mathematics is the Euler characteristic of a complex.
2. Every operation in mathematics lifts to an operation between complexes.
3. Every identity in mathematics remains true up to homotopy.

I. Frenkel

Local differentials:

Khovanov−Rozansky, arXiv/math.QA/0401268

where

Matrix factorizations:

$$D = \begin{pmatrix} 0 & A \\ B & 0 \end{pmatrix}$$

$$AB = BA = \omega I$$

A category, with "complexes", morphisms, homotopies, direct sums and tensor products.

D. Eisenbud