Summarizing Method of Variation of Parameters, section 3.6

To solve inhomogeneous equation

$$Ly := y'' + a_1 y' + a_0 y = f(t) \tag{1}$$

find a general solution $z = C_1 z_1 + C_2 z_2$ of the homogeneous equation

$$z'' + a_1 z' + a_0 z = 0 \tag{2}$$

with arbitrary constant coefficients C_1 and C_2. Then $y = C_1 z_1 + C_2 z_2$ with C_1 and C_2 functions satisfying $C_1' z_1 + C_2' z_2 = 0$ solves (1) **iff**

$$\begin{cases} C_1' z_1 + C_2' z_2 = 0, \\ C_1' z_1' + C_2' z_2' = f \end{cases} \tag{3}$$

or in the matrix form

$$\begin{pmatrix} z_1 & z_2 \\ z_1' & z_2' \end{pmatrix} \begin{pmatrix} C_1' \\ C_2' \end{pmatrix} = \begin{pmatrix} 0 \\ f \end{pmatrix} \tag{4}$$

(remember, for the first order linear ODEs we did the same albeit there was just one constant).