Graduate Studies in Mathematics
2021-22
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INTRODUCTION

The purpose of this handbook is to provide information about the graduate programs of the Department of Mathematics, University of Toronto. It includes detailed information about the department, its faculty members and students, a listing of core courses offered in 2021-2022, a summary of research activities, admissions requirements, application procedures, fees and financial assistance, and information about similar matters of concern to graduate students and prospective graduate students in mathematics.

This handbook is intended to complement the calendar of the university’s School of Graduate Studies, where full details on fees and general graduate studies regulations may be found.

For further information, please contact:

The Graduate Office  
Department of Mathematics  
University of Toronto  
40 St George St, Room 6166  
Toronto, Ontario, Canada M5S 2E4  
Telephone: (416) 978-7894  
Fax: (416) 978-4107  
Email: gradinfo@math.toronto.edu  
Website: http://www.math.utoronto.ca/cms/graduate-program/
1. DEPARTMENT OF MATHEMATICS

Mathematics has been taught at the University of Toronto since 1827. Since the first Canadian Ph.D. degree in mathematics was conferred to Samuel Beatty (under the supervision of John Charles Fields) in 1915, more than 400 Ph.D. degrees and 1,000 Master’s degrees have been awarded in this University. Many of our recent graduates are engaged in university teaching and a significant number of them hold administrative positions in universities or in the professional communities. Others are pursuing careers in industry (technological or financial), and in government.

The Department of Mathematics, University of Toronto is a distinguished faculty of more than sixty mathematicians. We have a large selection of graduate courses and seminars, and a diverse student body of domestic and international students, yet classes are small and the ratio of graduate students to faculty is low. We are in a unique position to take maximum advantage of the presence of the Fields Institute, which features special programs in pure and applied mathematics. Currently the Department has 167 graduate students, of whom 24 are enrolled in the Master’s program, 143 in the Ph.D. program.

Opportunities for graduate study and research are available in most of the main fields of pure and applied mathematics. These fields include real and complex analysis, ordinary and partial differential equations, harmonic analysis, nonlinear analysis, several complex variables, functional analysis, operator theory, C*-algebras, ergodic theory, group theory, analytic and algebraic number theory, Lie groups and Lie algebras, automorphic forms, commutative algebra, algebraic geometry, singularity theory, differential geometry, symplectic geometry, classical synthetic geometry, algebraic topology, set theory, set theoretic topology, mathematical physics, fluid mechanics, probability, optimal transportation, combinatorics, optimization, control theory, dynamical systems, computer algebra, cryptography, and mathematical finance.

We offer a research-oriented Ph.D., and Master’s program. Very strong students may be admitted directly to the Ph.D. program with a Bachelor’s degree; otherwise; it is normal to do a 1-year Master’s degree first. (Provisional admission to the Ph.D. program may be granted at the time of admission to the Master’s program.) The Master’s program may be extended to 16 months or 24 months for students who do not have a complete undergraduate preparation, or for industrial students engaged in a project.

There is a separate Master’s of Mathematical Finance Program not directly under the Department’s jurisdiction, with which some of our faculty members are associated.

During their studies here, graduate students are encouraged to participate in the life of the close community of U of T Mathematics. Almost all of them do some work in connection with undergraduate teaching, either as tutorial leaders, markers, or, especially in later years of their program, instructors. There is also a Mathematics Graduate Student Association, which organizes social and academic events and makes students feel welcome.
Graduate Faculty Members

AKCOGLU, M.A. (Professor Emeritus) Ph.D. 1963 (Brown)
- Ergodic theory, functional analysis, harmonic analysis
ALEXAKIS, Spyros (Professor) Ph.D. 2005 (Princeton)
- Geometric analysis and general relativity
ARETAKIS, S. (Assistant Professor) Ph.D. 2012 (University of Cambridge)
- Differential Geometry, Analysis of PDEs, General Relativity
ARTHUR, J. (University Professor, Mossman Chair) B.Sc. 1966 (Toronto), M.Sc. 1967 (Toronto), Ph.D. 1970 (Yale)
- Representations of Lie groups, automorphic forms
BARBEAU, E. (Professor Emeritus) M.A. 1961 (Toronto), Ph.D. 1964 (Newcastle)
- Functional analysis, optimization under constraint, history of analysis, number theory
- Theory of quantum invariants of knots, links and three manifolds
BIERSTONE, E. (Professor) B.Sc. 1969 (Toronto), Ph.D. 1973 (Brandeis)
- Singularity theory, analytic geometry, differential analysis
BINDER, I. (Associate Professor) Ph.D. 1997 (Caltech)
- Harmonic and complex analysis, conformal dynamics
BLAND, J. (Professor) Ph.D. 1982 (UCLA)
- Several complex variables, differential geometry
BLOOM, T. (Professor Emeritus) Ph.D. 1965 (Princeton)
- Several complex variables
BRAVerman, A. (Professor) Ph.D. 1998 (Tel Aviv)
- Representation theory, algebraic geometry
BREMER, J. (Assistant Professor) B.S. 2001 (Maryland), Ph.D. 2007 (Yale)
- Numerical analysis, numerical solution of differential and integral equations
BURCHARD, A. (Professor) Ph.D. (Georgia Tech) 1994
- Functional analysis
CHoi, M.-D. (Professor Emeritus) M.Sc. 1970 (Toronto), Ph.D. 1973 (Toronto)
- Operator theory, operator algebras, matrix theory
- Operators on Hilbert spaces, matrix theory and applications (including numerical analysis)
DAUVERGNE, D. (Assistant Professor) M.Sc. 2015 (Toronto), Ph.D. 2019 (Toronto)
- Probability, combinatorial probability, last passage percolation, KPZ universality, interacting particle systems, sorting networks, random polynomials, potential theory, random constraint satisfaction
DE SIMOI, J. (Assistant Professor) Ph.D. 2009 (University of Maryland)
- Stochastic and ergodic properties of smooth and piecewise smooth dynamical systems
DERZKO, N. (Associate Professor Emeritus) B.Sc. 1970 (Toronto), Ph.D. 1965 (Caltech)
- Functional analysis, structure of differential operators, optimization and control theory with applications to economics
ELLERS, E. (Professor Emeritus) Dr.rer.nat. 1959 (Hamburg)
- Classical groups
ELLIOTT, G. A. (Canada Research Chair and Professor) Ph.D. 1969 (Toronto)
- Operator algebras, K-theory, non-commutative geometry and topology
FRIEDLANDER, J. (University Professor) B.Sc. 1965 (Toronto), Ph.D. 1972 (Penn State)
• Analytic number theory
• Spectral theory of Schroedinger operators and localization
GRAHAM, I. (Professor Emeritus) B.Sc. 1970 (Toronto), Ph.D. 1973 (Princeton)
• Several complex variables, one complex variable
GROECHENIG, M. (Assistant Professor) D.Phil 2013 (University of Oxford)
• Higgs bundles and Hitchin systems, Algebraic K Theory, adeles, n-local fields, p-adic, motivic integration
GUALTIERI, M. (Professor) Ph.D. 2003 (Oxford)
• Differential geometry and mathematical physics
HALPERIN, S. (Professor Emeritus) M.Sc. 1966 (Toronto), Ph.D. 1970 (Cornell)
• Homotopy theory and loop space homology
HASLHOFER, R. (Assistant Professor) Ph.D. 2012 (ETH Zürich)
• Geometric analysis, differential geometry, partial differential equations
HERZIG, F. (Associate Professor) Ph.D. 2006 (Harvard)
• Number theory, Galois representations, automorphic forms
IVRII, V. (Professor) Ph.D. 1973 (Novosibirsk)
• Partial differential equations
JEFFREY, L. (Professor) Ph.D. 1992 (Oxford)
• Symplectic geometry, geometric applications of quantum field theory
JERRARD, Robert (Professor) Ph.D. 1994 (Berkeley)
• Nonlinear partial differential equations, Ginzburg-Landau theory
JURDJJEVIC, V. (Professor Emeritus) Ph.D. 1969 (Case Western)
• Systems of ordinary differential equations, control theory, global analysis
KAMNITZER, Joel (Professor) Ph.D. 2005 (Berkeley)
• Geometric and combinatorial representation theory
KAPOVITCH, V. (Professor) Ph.D. 1997 (University of Maryland)
• Global Riemannian geometry
KARSHON, Y. (Professor) Ph.D. 1993 (Harvard)
• Equivariant symplectic geometry
KHANIN, K. (Professor) Ph.D. 1983 (Landau Institute, Moscow)
• Dynamical systems and statistical mechanics
Khesin, B. (Professor) Ph.D. 1989 (Moscow State)
• Poisson geometry, integrable systems, topological hydrodynamics
KHOVANSKII, A. (Professor) Ph.D. 1973, Doctorat d’Etat 1987 (Steklov Institute, Moscow)
• Algebra, geometry, theory of singularities
KIM, Henry (Professor) Ph.D. 1992 (Chicago)
• Automorphic L-functions, Langlands’ program
KOPPARTY, S. (Assistant Professor) Ph.D. 2010 (MIT)
• Theory of computing, error-correcting codes, complexity theory, combinatorics, finite fields, randomness, and pseudorandomness
KUDLA, S. (Canada Research Chair and Professor) Ph.D. 1971 (Harvard)
• Automorphic forms, Arithmetic geometry and Theta functions
KUPERS, A. (Assistant Professor) Ph.D. (Stanford University)
• Algebraic and geometric topology, homotopy theory, manifolds, algebraic K-theory.
LANDON, B. (Assistant Professor) M.Sc. 2013 (McGill), Ph.D. 2018 (Harvard)
• Probability, random matrix theory
LIOKUMOVICH, Y. (Assistant Professor) Ph.D. 2015 (University of Toronto)
- Geometric Analysis, Metric Geometry
LORIMER, J.W. (Professor Emeritus) Ph.D. 1971 (McMaster)
- Rings and geometries, topological Klingenberg planes, topological chain rings
MARCOLLI, M. (Professor) Ph.D. 1997 (University of Chicago)
- Mathematical and theoretical physics, differential and algebraic geometry and topology,
  computational linguistics
McCANN, R. (Professor) Ph.D. 1994 (Princeton)
- Mathematical physics, mathematical economics, inequalities, optimization, partial
  differential equations
McCOOL, J. (Professor Emeritus) Ph.D. 1966 (Glasgow)
- Infinite group theory
MEINRENKEN, E. (Professor) Ph.D. 1994 (Universität Freiburg)
- Symplectic geometry
MENDELSOHN, E. (Professor Emeritus) Ph.D. 1968 (McGill)
- Block designs, combinatorial structures
MILMAN, P. (Professor) Ph.D. 1975 (Tel Aviv)
- Singularity theory, analytic geometry, differential analysis
MURASUGI, K. (Professor Emeritus) D.Sc. 1960 (Tokyo)
- Knot theory
MURNAGHAN, F. (Professor) Ph.D. 1987 (Chicago)
- Harmonic analysis and representations of p-adic groups
- Number theory
NABUTOVSKY, A. (Professor) Ph.D. 1992 (Weizmann Institute of Science)
- Geometry and logic
NACHMAN, A. (Professor) Ph.D. 1980 (Princeton)
- Inverse problems, partial differential equations, medical imaging
PANCHENKO, D. (Professor) Ph.D. 2002 (University of New Mexico)
- Applied probability
PAPYAN, V. (Assistant Professor) Ph.D. 2017 (Technion - Israel Institute of Technology)
- Deep Learning, Machine Learning, Data Science, Signal Processing
PUGH, M. (Professor) Ph.D. 1993 (Chicago)
- Scientific computing, nonlinear PDEs, fluid dynamics, computational neuroscience
PUSATERI, F. (Assistant Professor) Ph.D. 2011 (New York University)
- Partial differential equations, fluid dynamics, harmonic analysis and applications,
  Hamiltonian dynamics and small divisors
QUASTEL, J. (Professor) Ph.D. 1990 (Courant Institute)
- Probability, stochastic processes, partial differential equations
RAFI, K. (Professor) Ph.D. 2001 (Stony Brook)
- Teichmüller space, geometric group theory, hyperbolic geometry
REPKA, J. (Professor) B.Sc. 1971 (Toronto), Ph.D. 1975 (Yale)
- Group representations, automorphic forms
ROSENTHAL, P. (Professor Emeritus) Ph.D. 1967 (Michigan)
- Operators on Hilbert spaces
ROSSMAN, B. (Assistant Professor) Ph.D. 2010 (MIT)
- Complexity theory and logic
ROTMAN, R. (Professor) Ph.D. 1998 (SUNY, Stony Brook)
• Riemannian geometry

SARAF, S. (Assistant Professor) B.S. 2007 (MIT), M.S. 2009 (MIT), Ph.D. 2011 (MIT)
• Theoretical computer science, discrete mathematics, complexity theory, algebraic computation, coding theory, discrete geometry

SERKH, K. (Assistant Professor) Ph.D. 2016 (Yale)
• Numerical analysis, scientific computing, partial differential equations

SCHERK, J. (Associate Professor) D.Phil. 1978 (Oxford)
• Algebraic geometry

SECO, L. (Professor) Ph.D. 1989 (Princeton)
• Harmonic analysis, mathematical physics, mathematical finance

• Algebraic topology

SEN, D.K. (Professor Emeritus) Dr.es.Sc. 1958 (Paris)
• Relativity and gravitation, mathematical physics

SHANKAR, A. (Assistant Professor) Ph.D. 2012 (Princeton University)
• Number theory

SHARPE, R. (Professor Emeritus) B.Sc. 1965 (Toronto), M.Sc. 1966 (Toronto), Ph.D. 1970 (Yale)
• Differential geometry, topology of manifolds

SHLAPENTOKH-ROTHMAN, Y. (Assistant Professor) Ph.D. 2015 (MIT)
• Partial differential equations, general relativity, geometric analysis

SIGAL, I.M. (University Professor, Norman Stuart Robertson Chair in Applied Math) Ph.D. 1975 (Tel Aviv)
• Mathematical physics

• Fluid mechanics, particularly boundary layer theory

STINCHCOMBE, A. (Assistant Professor) Ph.D. 2013 (Courant Institute of Mathematical Sciences)
• Mathematical biology, scientific computing

• Partial differential equations, nonlinear analysis, numerical computations in fluid dynamics

TALL, F.D. (Professor Emeritus) Ph.D. 1969 (Wisconsin)
• Set theory and its applications, set-theoretic topology

TANNY, S.M. (Associate Professor Emeritus) Ph.D. 1973 (M.I.T.)
• Combinatorics, mathematical modeling in the social sciences

TIOZZO, G. (Assistant Professor) Ph.D. 2013 (Harvard)
• Dynamical systems and ergodic theory

TODORCEVIC, S. (Canada Research Chair and Professor) Ph.D. 1979 (Belgrade)
• Set theory and combinatorics

TSIMERMAN, J. (Assistant Professor) Ph.D. 2011 (Princeton University)
• Analytic number theory, Abelian varieties

UNGER, S. (Assistant Professor) Ph.D. 2013 (Carnegie Mellon)
• Set theory, Measurable Combinatorics

URIARTE-TUERO, I. (Associate Professor) Ph.D. 2004 (Yale)
• Harmonic and Complex Analysis
VIRAG, B. (Canada Research Chair and Professor) Ph.D. 2000 (Berkeley)
• Probability
WEISS, W. (Professor) M.Sc. 1972 (Toronto), Ph.D. 1975 (Toronto)
• Set theory, set-theoretic topology
YAMPOLSKY, M. (Professor) Ph.D. 1997 (SUNY, Stony Brook)
• Holomorphic and low-dimensional dynamical systems
YUEN, H. (Assistant Professor) Ph.D. 2016 (M.I.T)
• Quantum computing, complexity theory, cryptography, information theory
ZHANG, K. (Assistant Professor) Ph.D. 2007 (Pennsylvania State University)
• Dynamical systems: Hamiltonian dynamics, weak KAM theory, Arnold diffusion, smooth dynamics.
YU, W. (Assistant Professor) Ph.D. 2017 (M.I.T)
• Applied Mathematics, Computational Biology

2. THE GRADUATE PROGRAM

The Department of Mathematics offers graduate programs leading to Master of Science (M.Sc.) and Doctor of Philosophy (Ph.D.) degrees in mathematics, in the fields of pure mathematics and applied mathematics. Students admitted to our M.Sc. program are admitted either as “terminal master’s” students or as “doctoral stream” students, the latter implying the intent of continuing on to our Ph.D. program. Students admitted to the doctoral-stream master’s program are fully funded for one year at the master’s level. Funding is limited to four years at the Ph.D. level.

The M.Sc. Program

The M.Sc. program may be done on either a full- or part-time basis. Full-time students normally complete the program in one full year of study; three years in some cases. Part-time students may take up to six years to complete the program. The degree requirements are as follows:

1a. Completion of 6 half-courses (or the equivalent combination of half- and full-year courses). A current listing is available from the mathematics department website. The normal course load for full-time graduate students is 3 courses in the fall term and 3 in the spring term. Doctoral-stream students are required to take four half-course credits in core material.

1b. Completion of the Supervised Research Project (MAT 4000Y). This project is intended to give the student the experience of independent study in some area of advanced mathematics, under the supervision of a faculty member. The supervisor and the student, with the approval of the graduate coordinator decide the topic and program of study. The project is normally undertaken during the summer session, after the other course requirements have been completed, and has a workload roughly equivalent to that of a full-year course.

2. M.Sc. Thesis Option (less common than option 1). Students who take this option will be required to take and pass four half-courses and submit an acceptable thesis which should consist of a minimum of 20-pages of scholarly work. A presentation of the thesis results, in the form of a seminar, is required.
**The Ph.D. Program**

The Ph.D. program normally takes four years of full-time study beyond the Master’s level to complete. A Master’s degree is a prerequisite. Expected progress in the program is outlined in the following table:

<table>
<thead>
<tr>
<th>Year 1</th>
<th>Course work (6 courses); Pass at least 3 comprehensive exams; Select a thesis advisor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2</td>
<td>Supervisory committee selected by student and thesis advisor; First annual supervisory committee progress report due.</td>
</tr>
<tr>
<td>Year 3</td>
<td>Presentation of preliminary thesis results to supervisory committee; Achieve Ph.D. candidacy.</td>
</tr>
<tr>
<td>Year 4</td>
<td>Thesis Content Seminar; Departmental Ph.D. Thesis Examination; Final Ph.D. Thesis Examination at the School of Graduate Studies.</td>
</tr>
<tr>
<td>Year 4 – October</td>
<td>Students interested in academic employment after the Ph.D. must have major thesis results ready.</td>
</tr>
</tbody>
</table>

**The Ph.D. Direct-Entry Program**

The Ph.D. Direct-Entry program normally takes five years of full-time study to complete. Expected progress in the program is outlined in the following table:

<table>
<thead>
<tr>
<th>Year 1</th>
<th>6 courses of which four half-course credits in core material. Completion of a summer Supervised Research Project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 2</td>
<td>Course work; Pass at least 3 comprehensive exams; Select a thesis advisor.</td>
</tr>
<tr>
<td>Year 3</td>
<td>Supervisory committee selected by the supervisor and the student; First annual supervisory committee progress report due.</td>
</tr>
<tr>
<td>Year 4</td>
<td>Presentation of preliminary thesis results to supervisory committee; Achieve Ph.D. candidacy*.</td>
</tr>
<tr>
<td>Year 5</td>
<td>Thesis Content Seminar; Departmental Ph.D. Thesis Examination; Final P Ph.D. Thesis Examination at the School of Graduate Studies.</td>
</tr>
<tr>
<td>Year 5 – October</td>
<td>Students interested in academic employment after the Ph.D. must have major thesis results ready.</td>
</tr>
</tbody>
</table>

1. **Coursework for Ph.D. (4-year program):**

Completion of at least 6 half-courses (or the equivalent combination of half- and full-year courses). A current listing is available from the mathematics department website. Normally, 6 half-courses are taken in the first year of study (3 half-courses in the fall term and 3 in the spring term). It is strongly recommended that the student take some additional courses in later years. Effective September 2020, students must achieve an A- grade in each core course. Furthermore, students can choose to take any combination of core courses from the 12 core courses option i.e. students can choose to take MAT1100H but not MAT1101H, MAT1060H but not MAT1061H etc...).
2. **Coursework for Ph.D. (Direct-Entry program):**

Completion of at least 8 half-courses (or the equivalent combination of half- and full-year courses). A current listing is available from the mathematics department website. Normally, 6 half-courses are taken in the first year of study (3 half-courses in the fall term and 3 in the spring term) and 2 half-courses in the second year. It is strongly recommended that the student do a Supervised Reading Project (MAT1900Y, MAT1950Y) in the summer between Year 1 and 2 and take some additional courses in other years. Students must achieve an A- grade in each core course.

3. **Comprehensive Examination:**

The student is required to pass at least three comprehensive examinations in basic mathematics before beginning an area of specialization. The examinations in the six general areas (analysis (real and complex), algebra, topology, partial differential equations, linear algebra & optimization and probability) take place over a 2-day period in early or late September. Passing the comprehensive exam will fulfill the core course requirement.

Effective September 2021, there will be a single "pass/fail" comprehensive exam that will consist of 12 problems from the different subject areas named above. Correctly solving 6 problems out of the 12 constitutes a "pass".

Syllabi for the pure mathematics comprehensive exams appear in Appendix A. Copies of mock examination questions and/or past written examination papers are accessible to all candidates [here](#).

Students with interests in applied mathematics or with Physics should refer to Appendix B for possible alternate comprehensive exams.

Core courses requirement is to be satisfied within 13 months of entering the Ph.D. program unless the Examination Committee grants permission in writing for a deferral. Ph.D. candidacy must be achieved by the end of the 3rd year.

**Supervisory Committee:** Ph.D. students must select a supervisor by the beginning of their second year in the Ph.D. program. In accordance with School of Graduate Studies’ regulations, a supervisory committee (SC) will be established for each Ph.D. student who has chosen a research area and a supervisor. This committee consists of three faculty members including the supervisor. The SC is expected to meet with the student at least annually, including on the last year of studies. Since some of the role of the SC is private and confidential, it is not appropriate to substitute these meetings with public lectures. The SC will file an annual written report with the graduate office.

The purpose of the Supervisory Committee (SC) is to monitor the student’s progress at least on an annual basis, keeping the following in mind.

No supervisor is perfect! The SC may be able to offer further mathematical and further career advice beyond what the supervisor alone may offer. This is relevant both at the start of studies, when mathematical advice is most in need, and towards the end of studies, when career advice is needed.
No student is perfect! The SC should note if a student is falling behind and should propose ways for the student to catch up, if necessary.

While most student-supervisor relationships are cordial and productive, occasional misunderstandings, miscommunications and cases of false expectations do occur. The SC should note if a student-supervisor relationship is heading wrong and make sure that steps are taken to fix the problems.

Further information about general graduate supervision is available [here](#).

2. **Thesis**: The main requirement of the degree is an acceptable thesis. This will embody an individual contribution to original research of a standard that warrants publication in the research literature. It must be written under the supervision of one or more members of the department. The student presents the thesis results in three stages.

   (i) *Thesis Content Seminar*. This is an opportunity for the student to present his/her thesis results to department members. The presentation frequently takes place within one of the regular departmental research seminars.

   (ii) *Departmental Oral Examination*. The student gives a 20-minute summary of the thesis and must defend it before a departmental examination committee. Copies of the thesis should be available two weeks before the departmental oral examination. The committee may approve the thesis without reservations, or approve the thesis on condition that revision be made, or require the student to take another departmental oral examination.

   (iii) *Final Oral Examination*. Eight weeks after the successful completion of the departmental oral, the student proceeds to the final oral examination conducted by the School of Graduate Studies. The thesis is sent to an external reader who submits a report two weeks prior to the examination; this report is circulated to members of the examination committee and to the student. The examination committee consists of four to six faculty members; it is recommended that the external reader attend the examination. The student gives a 20-minute summary of the thesis, which is followed by a question period.

As an alternative to the above examination procedure, it is possible, on the Ph.D. candidate's initiative and with the agreement of the candidate's academic advisor, to substitute a 50-minute presentation for the 20-minute presentation in the departmental oral examination and the final oral examination. The department will attempt to accommodate such requests though technical and administrative considerations (space and scheduling issues, communication issues when some committee members are not physically present, compliance issues with SGS rules, etc.) may prevent this alternative. It is recommended that such requests will be made at least 4 weeks in advance of the departmental examination.

3. **Students** are expected to become extensively involved in departmental life (seminars, colloquia and related activities).
3. ADMINISTRATION OF THE GRADUATE PROGRAM

A central administration authority called the School of Graduate Studies establishes the basic policies and procedures governing all graduate study at the University of Toronto. Detailed information about the School is obtained in its calendar.

The Department of Mathematics has its own graduate administrative body—the graduate committee—composed of 12-15 faculty members appointed by the Chair of the Department, and five graduate students elected by the Mathematics Graduate Students Association. One of the faculty members is the Graduate Coordinator, who is responsible for the day-to-day operations of the program. The graduate committee meets frequently throughout the year to consider matters such as admissions, scholarships, course offerings, and departmental policies pertaining to graduate students. Student members are not permitted to attend meetings at which the agenda concerns confidential matters relating to other students. Information regarding appeals of academic decisions is given in the Grading Procedures section of the Calendar of the School of Graduate Studies. Students may also consult the Graduate Coordinator (or the student member of the departmental Graduate Appeals Committee) regarding information about such appeals.

General Outline of the 2021-22 Academic Year

Registration  Monday, July 19, 2021 – Friday, September 10, 2021
Fall Term    Classes begin Thursday, September 9, 2021
Spring Term  Classes begin Monday, January 3, 2022

Official Holidays (University Closed):

<table>
<thead>
<tr>
<th>Holiday</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Labour Day</td>
<td>Monday, September 6, 2021</td>
</tr>
<tr>
<td>Thanksgiving Day</td>
<td>Monday, October 11, 2021</td>
</tr>
<tr>
<td>Winter Break</td>
<td>Wednesday, December 22, 2021 – Sunday, January 2, 2022 (inclusive)</td>
</tr>
<tr>
<td>Family Day</td>
<td>Monday, February 21, 2022</td>
</tr>
<tr>
<td>Good Friday</td>
<td>Friday, April 15, 2022</td>
</tr>
<tr>
<td>Victoria Day</td>
<td>Monday, May 23, 2022</td>
</tr>
<tr>
<td>Presidential Day</td>
<td>Thursday, June 30, 2022</td>
</tr>
<tr>
<td>Canada Day</td>
<td>Friday, July 1, 2022</td>
</tr>
<tr>
<td>Civic Holiday</td>
<td>Monday, August 1, 2022</td>
</tr>
</tbody>
</table>
4. GRADUATE COURSES

The following is a list and description of the core courses offered to graduate students in the 2021-22 academic year. These are the basic beginning graduate courses. They are designed to help the student broaden and strengthen his/her general background in mathematics prior to specializing towards a thesis. A student with a strong background in the area of any of the core courses should not take that particular course. A complete listing of all graduate courses offered in a given year is available from the mathematics department website. These include cross-listed graduate courses and topics courses. In addition, graduate students may take several intermediate (300-level) undergraduate courses (listed in the Faculty of Arts and Science Calendar) if their background is felt to be weak in some area; no graduate course credit is given for these courses.

There are three other means by which graduate students may obtain course credit, apart from completing the formal courses listed on the following pages. In each of these cases, prior approval of the graduate coordinator is required.

1. Students may take a suitable graduate course offered by another department. Two-thirds of the course requirements for each degree should be in the Mathematics Department.
2. It is sometimes possible to obtain course credit for appropriately extensive participation in a research seminar (see Research Activities section).
3. It is also possible to obtain a course credit by working on an individual reading course under the supervision of one of the faculty members, provided the material covered is not available in one of the formal courses or research seminars. (Note: this is distinct from the MAT 4000Y Supervised Research Project required of M.Sc. students.)

Most courses meet for three hours each week, either in three one-hour sessions or two longer sessions. For some courses, particularly those cross-listed with undergraduate courses, the times and locations of classes will be set in advance of the start of term. For other courses, the times and locations of classes will be established at organizational meetings during the first week of term, so that a time convenient for all participants may be arranged. During registration week, students should consult the math department website for class and organization meeting times and locations.
**CORE COURSES**

**MAT 1000HF (MAT 457H1F)**  
**REAL ANALYSIS I**  
Y. Shlapentokh-Rothman

**Measure Theory**: Lebesgue measure and integration, convergence theorems, Fubini's theorem, Lebesgue differentiation theorem, abstract measures, Caratheodory extension theorem, Radon-Nikodym theorem.

**Functional Analysis**: Hilbert spaces, orthonormal bases, Riesz representation theorem, operators, $L^p$-spaces, Holder and Minkowski inequalities.

**Textbook**:  

**References**:  
Elias Stein and Rami Shakarchi, Measure Theory, Integration, and Hilbert Spaces  
Elliott H. Lieb and Michael Loss, Analysis, AMS Graduate Texts in Mathematics, 14 (either edition)  
H. L. Royden, Real Analysis, Macmillan, 1998  
A. N. Kolmogorov and S. V. Fomin: Introductory Real Analysis, 1975

**MAT 1001HS (MAT 458H1S)**  
**REAL ANALYSIS II**  
A. Burchard

**Fourier analysis**: Fourier series and transforms, Fourier inversion and Plancherel formula, estimates and convergence results, topological vector spaces, Schwartz space, distributions.

**Functional Analysis**: The main topic here will be the spectral theorem for bounded self-adjoint operators, possibly together with its extensions to unbounded and differential operators

**Textbook**:  

**References**:  
E. Lorch, Spectral Theory.  

**MAT 1002HS (MAT 454H1S)**  
**COMPLEX ANALYSIS**  
M. Goldstein

2. Harmonic functions. Poisson’s integral formula and Dirichlet’s problem
3. Conformal mapping, Riemann mapping theorem.
4. Analytic continuation, Monodromy Theorem, Riemann surfaces.
5. Modular functions and the Picard Theorems.
6. Other topics are possible, like product theorems, elliptic functions, and non-isolated removability theorems.

**Recommended prerequisites:** A first course in complex analysis and a course in real analysis. Measure theory is not required.

**Main References:**
L. Ahlfors: Complex Analysis, 3rd Edition  
Stein and Shakarchi: Complex Analysis

**Additional References:**
T. Gamelin, Complex Analysis  
W. Rudin, Real and Complex Analysis, 2nd or 3rd edition  
D. Sarason, Complex Function Theory

**MAT 1060HF**  
**PARTIAL DIFFERENTIAL EQUATIONS I**  
**R. McCann**

This is a basic introduction to partial differential equations as they arise in physics, geometry and optimization. It is meant to be accessible to beginners with little or no prior knowledge of the field. It is also meant to introduce beautiful ideas and techniques, which are part of most analysts’ basic bag of tools. A key theme will be the development of techniques for studying non-smooth solutions to these equations.

**Textbook:**

**References:**


**MAT 1061HS**  
**PARTIAL DIFFERENTIAL EQUATIONS II**  
**I. M. Sigal**

This course will consider a range of mostly nonlinear partial differential equations, including elliptic and parabolic PDE, as well as hyperbolic and other nonlinear wave equations. In order to study these equations, we will develop a variety of methods, including variational techniques, and fixed point theorems. One important theme will be the relationship between variational questions, such as critical Sobolev exponents, and issues related to nonlinear evolution equations, such as finite-time blowup of solutions and/or long-time asymptotics.

The prerequisites for the course include familiarity with Sobolev and other function spaces, and in
particular with fundamental embedding and compactness theorems.

Other topics in PDE will also be discussed.

Textbook:

References:


MAT 1100HF
ALGEBRA I
I. Varma


Group Theory: Isomorphism theorems, group actions, Jordan-Hölder theorem, Sylow theorems, direct and semidirect products, finitely generated abelian groups, simple groups, symmetric groups, linear groups, nilpotent and solvable groups, generators and relations.

Ring Theory: Rings, ideals, Euclidean domains, principal ideal domains, and unique factorization domains.

Modules: Modules and algebras over a ring, tensor products, modules over a principal ideal domain.

Recommended prerequisites are a full year undergraduate course in Linear Algebra and one term of an introductory undergraduate course in higher algebra, covering, at least, basic group theory. While this material will be reviewed in the course, it will be done at "high speed", assuming that you have already some familiarity with the basics. You will be very well prepared indeed, if you have no difficulties reading and understanding the book, listed here under "Other References", M. Artin: Algebra that the author wrote for his undergraduate algebra courses at MIT.

Textbooks:
Dummit and Foote: Abstract Algebra, 2nd Edition

Other References:
Jacobson: Basic Algebra, Volumes I and II.
Cohn: Basic Algebra
M. Artin: Algebra.
MAT 1101HS
ALGEBRA II
F. Herzig

**Fields:** Algebraic and transcendental extensions, normal and separable extensions, fundamental theorem of Galois theory, solution of equations by radicals.

**Commutative Rings:** Noetherian rings, Hilbert basis theorem, invariant theory, Hilbert Nullstellensatz, primary decomposition, affine algebraic varieties. Structure of semisimple algebras, application to representation theory of finite groups.

**Textbooks:**
Dummit and Foote: Abstract Algebra, 3rd Edition

**Other References:**
Jacobson: Basic Algebra, Volumes I and II.
Cohn: Basic Algebra
M. Artin: Algebra.

MAT 1300HF
TOPOLOGY I
Y. Liokumovich

Local differential geometry: the differential, the inverse function theorem, smooth manifolds, the tangent space, immersions and submersions, regular points, transversality, Sard’s theorem, the Whitney embedding theorem, smooth approximation, tubular neighborhoods, the Brouwer fixed point theorem.

Differential forms: exterior algebra, forms, pullbacks, integration, Stokes’ theorem, div grad curl and all, Lagrange’s equation and Maxwell’s equations, homotopies and Poincare’s lemma, linking numbers.

**Prerequisites:** Linear algebra; vector calculus; point set topology

**Textbook:**
John M. Lee: Introduction to Smooth Manifolds

MAT 1301HS
TOPOLOGY II
L. Jeffrey

Fundamental groups: paths and homotopies, the fundamental group, coverings and the fundamental group of the circle, Van-Kampen’s theorem, the general theory of covering spaces.

Homology: simplices and boundaries, prisms and homotopies, abstract nonsense and diagram chasing, axiomatics, degrees, CW and cellular homology, subdivision and excision, the generalized Jordan curve theorem, salad bowls and Borsuk-Ulam, cohomology and de-Rham’s theorem, products.
Textbook:
Allen Hatcher, Algebraic Topology

Recommended Textbooks:
Munkres, Topology
Munkres, Algebraic Topology

MAT1600HF
MATHEMATICAL PROBABILITY I
D. Dauvergne

The class will cover classical limit theorems for sums of independent random variables, such as the Law of Large Numbers and Central Limit Theorem, conditional distributions and martingales, metrics on probability measures.

Recommended prerequisite: Real Analysis I.

Textbook:

MAT1601HS
MATHEMATICAL PROBABILITY I
B. Landon

The class will cover some of the following topics: Brownian motion and examples of functional central limit theorems, Gaussian processes, Poisson processes, Markov chains, exchangeability.

Recommended prerequisites: Real Analysis I and Probability I.

Textbook:

MAT1850HF
LINEAR ALGEBRA AND OPTIMIZATION
M. Pugh

This course will develop advanced methods in linear algebra and introduce the theory of optimization. On the linear algebra side, we will study important matrix factorizations (e.g. LU, QR, SVD), matrix approximations (both deterministic and randomized), convergence of iterative methods, and spectral theorems. On the optimization side, we will introduce the finite element method, linear programming, gradient methods, and basic convex optimization. The course will be focused on fundamental theory, but appropriate illustrative applications may be chosen by the instructor.
INDIVIDUAL READING COURSES

Students requiring individual course numbers:

**MAT 1900Y/1901H/1902H**
**READING IN PURE MATHEMATICS**
Numbers assigned for students wishing individual instruction in an area of pure mathematics.

**MAT 1950Y/1951H/1952H**
**READING IN APPLIED MATHEMATICS**
Numbers assigned for students wishing individual instruction in an area of applied mathematics.

COURSE IN TEACHING TECHNIQUES

The following course is offered to help train students to become effective lecturers. It is not for degree credit and is not to be offered every year.

**MAT 1499HS**
**TEACHING LARGE MATHEMATICS CLASSES**
J. Repka

The goals of the course include techniques for teaching large classes, sensitivity to possible problems, and developing an ability to criticize one’s own teaching and correct problems.

Assignments will include such things as preparing sample classes, tests, assignments, course outlines, designs for new courses, instructions for teaching assistants, identifying and dealing with various types of problems, dealing with administrative requirements, etc.

The course will also include teaching a few classes in a large course under the supervision of the instructor. A video camera will be available to enable students to tape their teaching for later (private) assessment.

COURSES FOR GRADUATE STUDENTS FROM OTHER DEPARTMENTS

(Math graduate students cannot take the following courses for graduate credit.)

**MAT 2000Y** **READINGS IN THEORETICAL MATHEMATICS**
**MAT 2001H** **READINGS IN THEORETICAL MATHEMATICS I**
**MAT 2002H** **READINGS IN THEORETICAL MATHEMATICS II**

(These courses are used as reading courses for engineering and science students in need of instruction in special topics in theoretical mathematics. These course numbers can also be used as dual numbers for some third and fourth year undergraduate mathematics courses if the instructor agrees to adapt the courses to the special needs of graduate students. A listing of such courses is...
available in the 2021–2022 School of Graduate Studies Calendar. Students taking these courses should get an enrolment form from the graduate studies office of the Mathematics Department. Permission from the instructor is required.)

**PROFESSIONAL DEVELOPMENT PROGRAMS OFFERED BY SGS**

*GCAC Writing Centre*
The Graduate Centre for Academic Communication Writing Centre, at the School of Graduate Studies, offers one-on-one consultations to graduate students who seek individualized assistance with their writing. In these sessions, trained instructors work with you to improve your capacity to plan, write, and revise your academic assignments. Please note: The GCAC Writing Centre is not a proofreading service. Instructors do not edit for you; they teach you to revise and edit your own work. Information and registration: [https://www.sgs.utoronto.ca/resources-supports/gcac/writing-centre/](https://www.sgs.utoronto.ca/resources-supports/gcac/writing-centre/)

*Graduate Professional Skills Program (GPS)*
The Graduate Professional Skills program (GPS) is a new initiative from the School of Graduate Studies to help doctoral-stream graduate students become fully prepared for their future. It focuses on skills beyond those conventionally learned within a disciplinary program, skills that may be critical to success in the wide range of careers that graduates enter, both within and outside academia. GPS can help you to communicate more effectively, plan and manage your time, learn entrepreneurial skills, understand and apply ethical practices, and work effectively in teams and as leaders. The GPS consists of a range of optional “offerings” with a time commitment roughly equivalent to 60 hours of work. Its successful completion will be recognized by a transcript notation. For more information: [https://www.sgs.utoronto.ca/resources-supports/gpd/graduate-professional-skills-gps/#section_1](https://www.sgs.utoronto.ca/resources-supports/gpd/graduate-professional-skills-gps/#section_1)

*MITACS*
Mitacs ([www.mitacs.ca](http://www.mitacs.ca)) is a national research organization offering unique research and training programs to graduate students and postdoctoral fellows (PDFs) in Canada. For more information: [https://www.sgs.utoronto.ca/awards/mitacs-globalink-graduate-fellowship/](https://www.sgs.utoronto.ca/awards/mitacs-globalink-graduate-fellowship/)

**5. RESEARCH ACTIVITIES**

The Department of Mathematics offers numerous research activities, in which graduate students are encouraged to participate. Research seminars are organized informally at the beginning of each year by one or more faculty members and/or students, and are offered to faculty and graduate students on a weekly basis throughout the year. The level and specific content of these seminars varies from year to year, depending upon current faculty and student interest, and upon the availability and interests of invited guest lecturers. The following research seminars were offered in the past year:

- Algebra and Geometry Seminar
- Analysis and Applied Math Seminar
- Dynamics Seminar
- Fields Colloquium/Seminar in Applied Math
- Ganita Seminar
- Geometric Representation Theory Seminar
- Geometry and Topology Seminar
Graduate Student Seminar  
Homological Methods Seminar  
Inverse Problems and Image Analysis Seminar  
Number Theory/Representation Theory Seminar  
Operator Theory Seminar  
Probability, Geometry and Groups Learning Seminar  
Probability Study Group  
Student Number Theory Seminar  
Symplectic Seminar  
Teichmüller Theory and Dynamics Learning Seminar  
Toronto Probability Seminar  
Toronto Set Theory Seminar  
Trace Formula Working Seminar  
Women in Mathematics  
Working Group in Hamiltonian Systems Seminar  

A full list of events in the department can be found [here](#).

In addition to the weekly seminars, there are numerous special seminars throughout the year, a series of colloquia, and an active program of visiting lecturers:

Departmental Colloquium  
Blyth Lecture Series

Graduate students are also encouraged to attend lectures and seminars offered by other departments.

6. ADMISSION REQUIREMENTS AND APPLICATION PROCEDURES

Due to the large numbers of applications received in the Department of Mathematics each year, serious consideration will only be given to applicants with strong backgrounds in theoretical mathematics and with first class academic standing.

Application materials and admission requirements are available from the [Department of Mathematics website](#).

Please read all instructions carefully and note the deadlines. In addition, the Department of Mathematics requires three letters of reference. The letters must be from three people familiar with your mathematical work, giving their assessment of your potential for graduate study and research in mathematics.

It is essential that all incoming graduate students have a good command of English. Facility in the English language must be demonstrated by all applicants educated outside Canada whose primary language is not English. This requirement is a condition of admission and should be met before application. Here are three ways to satisfy this requirement: (1) Test of English as a Foreign Language (TOEFL): (a) internet-based test (iBT), minimum score of 22/30 for both the Writing and Speaking sections, with an overall minimum TOEFL score of 93/120, or (b) paper-based test, minimum score 580, with TWE (Test of Written English), minimum score 5.0; (2) a score of at least 85 on the Michigan English Language Assessment Batter (MELAB); (3) a score of at least 7.0 on the International English Language Testing Service (IELTS). Applicants are required to satisfy this
requirement by December 20, so that scores are available at the time applications are considered.

**Suggested prerequisites**

We recognize that our students come from many different places and with a significant range of differing backgrounds. Hence there is no fixed and rigid list of prerequisites, and applicants are considered and often admitted even if their formal previous mathematical education is very different from the informal list of prerequisites below. **In general, we’d like to see some sort of overall mathematical maturity and experience, and we appreciate (though we do not require) evidence of in-depth concentration in one mathematical discipline or another.**

Yet here is a non-binding list of courses that are recommended to applicants from within the University of Toronto in order to be seriously considered for the doctoral stream master’s program. Students coming from other institutions will have to make the appropriate substitutions:

2nd year Advanced ODE’s, e.g. MAT 267


Real Analysis, e.g. MAT 337


3rd year Real Analysis, e.g. MAT 357H


3rd year Complex Analysis, e.g. MAT 354H

Approximate syllabus: Complex numbers, the complex plane and Riemann sphere, Möbius transformations, elementary functions and their mapping properties, conformal mapping, holomorphic functions, Cauchy’s theorem and integral formula. Taylor and Laurent series, maximum modulus principle, Schwarz’s lemma, residue theorem and residue calculus.

Linear Algebra, e.g. MAT 224H


3rd year Algebra, e.g. MAT 347Y

Approximate syllabus: Groups, subgroups, quotient groups, Sylow theorems, Jordan-Hölder
theorem, finitely generated abelian groups, solvable groups. Rings, ideals, Chinese remainder theorem; Euclidean domains and principal ideal domains: unique factorization. Noetherian rings, Hilbert basis theorem. Finitely generated modules. Field extensions, algebraic closure, straight-edge and compass constructions. Galois theory, including insolvability of the quintic.

3\textsuperscript{rd} year Topology, e.g. MAT 327H


In addition to that we also value some ability in computer programming and some background in physics (though neither is required).

Groups and Symmetries, e.g. MAT 301H


Complex Variables, e.g. MAT 334H

Approximate syllabus: Theory of functions of one complex variable, analytic and meromorphic functions. Cauchy’s theorem, residue calculus, conformal mappings, introduction to analytic continuation and harmonic functions.

7. POLICY ON FINANCIAL SUPPORT, FEES AND FINANCIAL ASSISTANCE

Department of Mathematics Policy on Financial Support of Graduate Students

Ph.D. Students: At the time of admission to the Ph.D. program, students will normally be guaranteed support for a period of 4 to 5 years (five years in the case of students admitted directly from a Bachelor’s program), except that students who complete their degree requirements earlier will not be supported past the end of the academic year in which they finish. This guarantee will be made up of a mix of fellowships (including external awards such as NSERC, OGS), teaching assistantships, and other sources of funding, at the discretion of the Department; and is subject to satisfactory academic progress, the maintenance of good standing, and in the case of teaching assistantships, satisfactory performance in that role, as judged by the Department. Absent this, support may be reduced, suspended, or discontinued.

In exceptional circumstances, some funding may be provided to students in a subsequent year, but the Department expects that students will normally have completed their degree requirements within the four-year period.

M.Sc. Students: Students who are granted provisional admission to the Ph.D. program at the time of admission will receive financial support, for one year only.

All full-time students in the first or second year of a Master’s program are eligible for teaching assistant work (subject to availability and satisfactory performance).
Fees
Listed below are the fees for the 2021-22 academic session, including incidental fees and the health insurance premium for visa students. For detailed information, visit the School of Graduate Studies Fall/Winter Fee & Refund Schedules.

Domestic Fees

<table>
<thead>
<tr>
<th>Table A: Program Fee - Degree Students</th>
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<tbody>
<tr>
<td><strong>Doctoral &amp; Doctoral Stream - St. George Campus</strong></td>
</tr>
<tr>
<td><strong>PhD, MusD, SJD, MA, MASC, MSc, MScF, MusM (Music Ed.)</strong></td>
</tr>
<tr>
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<td><strong>2021-2022 Fall-Winter Session</strong></td>
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1) There may be additional ancillary fees levied for enrolment in specific courses or for individual circumstances. Check your ACORN invoice. Fall or Winter term incidental fees may be subject to change. The amounts listed on this schedule are the maximum amounts that will be billed.

Table B: Course Fee - Non-Degree, Special Students

| **Table 1** | **Course Fee - Non-Degree, Special Students** |
|-------------|
| **2021-2022 Fall-Winter Session** | **Course Load** | **Course Fees** | ** Mandatory Incidental, System Access & Ancillary Fees (Note 1)** | **Total** | **Notes** |
| 0.5 | $931.00 | $936.74 | $1,867.74 |
| 1.0 | $1,862.00 | $936.74 | $2,798.74 |

1) There may be additional ancillary fees levied for enrolment in specific courses or for individual circumstances. Check your ACORN invoice. Fall or Winter term incidental fees may be subject to change. The amounts listed on this schedule are the maximum amounts that will be billed.

International Fees

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2) Information about UHIP can be viewed at https://studentlife.utoronto.ca/cie/uhip.

Table B: Course Fee - Non-Degree, Special Students

| **Table 1** | **Course Fee - Non-Degree, Special Students** |
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| **2021-2022 Fall-Winter Session** | **Course Load** | **Course Fees** | ** Mandatory Incidental, System Access & Ancillary Fees (Note 1)** | **Univ. Health Insurance Plan (UHIP) Fees (Note 2)** | **Total** | **Notes** |
| 0.5 | $931.00 | $936.74 | $756.00 | $2,623.74 |
| 1.0 | $1,862.00 | $936.74 | $756.00 | $3,554.74 |

1) There may be additional ancillary fees levied for enrolment in specific courses or for individual circumstances. Check your ACORN invoice. Fall or Winter term incidental fees may be subject to change. The amounts listed on this schedule are the maximum amounts that will be billed.

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| Full-time: Fall - Winter | $26,210.00 | $1,769.99 | $756.00 | $28,735.99 |
| Full-time: Fall or Winter | $13,105.00 | $912.51 | $756.00 | $14,773.51 |
| Part-time: Fall - Winter | $7,863.00 | $936.74 | $756.00 | $9,555.74 |
| Part-time: Fall or Winter | $3,931.50 | $495.89 | $756.00 | $5,183.39 |

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- For detailed information, visit the School of Graduate Studies Fall/Winter Fee & Refund Schedules.

Fall/Winter Fee & Refund Schedules
Financial Assistance

Below is a list of those types of financial assistance most commonly awarded to mathematics graduate students in 2020-21. This information should also be applicable for students who wish to apply for the 2021-22 academic year; the deadlines for applications will be altered slightly in accordance with the 2021-22 calendar. Some awards are available from external funding agencies; others come from within the University.

Less common scholarships, offered by smaller or foreign funding agencies, are also available; information about these may be found here. Announcements are made to students.

Vanier Canada Graduate Scholarships (Vanier CGS) Program

Value: $50,000 per annum for a maximum of 3 years
Eligibility: a student must be nominated by a Canadian university. Vanier Scholarships must be held by the student at the university that nominated them. The scholarships are not transferable: they may not be taken to another university Applicants do not have to be registered as doctoral students at the time of application, but must be registered as doctoral students at a Canadian university when they take up the Vanier Scholarship. Domestic and international students are all eligible for Vanier Scholarships. Students who have held or are currently holding Tri-Agency scholarship funding for their doctoral program are not eligible to apply.

Application: https://www.researchnet-recherchenet.ca/rnr16/LoginServlet
2022 Scholarship Deadline: Friday, September 17, 2021

Natural Sciences and Engineering Research Council (NSERC) Postgraduate Scholarships and Canada Graduate Scholarships

Value: approx. $17,300-$35,000 for a twelve-month period
Eligibility: Canadian citizens, permanent residents; first class academic standing; full-time attendance
Application: apply through the university you are currently attending; application available at www.nserc.ca
Deadline: early October. Consult department for deadline

Ontario Graduate Scholarships (OGS)

Value: approx. $5,000 per term for two or three terms
Eligibility: no citizenship restrictions; first class academic standing; full-time attendance at an Ontario university
Application: apply through the department you wish to attend (https://www.sgs.utoronto.ca/awards/ontario-graduate-scholarship/).
Deadline: Consult department for deadline.

Queen Elizabeth II Graduate Scholarship in Science and Technology (QEII-GSST)

Value: approx. $15,000 for a twelve-month period
Eligibility: Canadian citizens, permanent residents; first class academic standing; full-time attendance

There may be additional ancillary fees levied for enrolment in specific courses or for individual circumstances. Check your ACORN invoice. Fall or Winter term incidental fees may be subject to change. The amounts listed on this schedule are the maximum amounts that will be billed.
University of Toronto Fellowships

**Value:** minimum $1,000

**Eligibility:** no citizenship restrictions; at least an A- average; full-time attendance at the University of Toronto

**Application:** graduate school applicants will automatically be considered

**Deadline:** early December

**Connaught International Scholarship for Doctoral Students** (entrance scholarship)

**Value:** The effective value awarded to each student will be $35,000.00 total (including tuition).

Scholarships will continue in the same form throughout the normal period in the funded cohort, provided progress.

**Application:** A graduate school application. Graduate units decide nominations in the winter semester.

**Deadline:** To be considered, international doctoral applicants must submit a graduate school application by early December

**Ontario Trillium Scholarship (OTS) for Doctoral Students** (entrance scholarship)

The Ontario Trillium Scholarship (OTS) is designed to attract international Ph.D. students to the University of Toronto.

**Value:** Each OTS will be worth $40,000 per annum (including tuition), and is renewable for four years, provided progress.

Funding support for new Ontario Trillium Scholarship offers is currently suspended. Existing award holders will continue to receive the balance of their renewable scholarship.

**Research Assistantships**

**Value:** a limited amount of funds is available for academically worthy students

**Eligibility:** no citizenship restrictions; full-time attendance; high academic standing

**Application:** graduate school applicants will be considered automatically

**Deadline:** early December

**Teaching Assistantships**

**Value:** $46.24 per hour; number of hours per week will not exceed a maximum average of 8

**Eligibility:** full-time students who are accepted by the Mathematics Department (subject to satisfactory performance); may be held in conjunction with other awards

**Application:** link to online TA application is provided within job posting at unit1.hrandequity.utoronto.ca

**Deadline:** for summer semester, March/April; for fall/winter semesters, July. Check unit1.hrandequity.utoronto.ca for relevant postings.
**Doctoral Thesis Completion Award (DCA)**
The Department of Mathematics receives funding from the university to help support students beyond the funded cohort: Doctoral Thesis Completion Award. The policy of the mathematics department is to spend its DCA funds aiming to bring the total amount of after-fees funding (including both external and other internal sources) for all fifth-year students who are in good academic standing (including a satisfactory supervisory committee report) to be as close to uniform within that group as feasible, and in as much as possible, close to the level of support funded-cohort students are receiving. Shall there be any DCA funds remaining after that, these funds will be distributed on a similar basis among sixth year students.

**Arts and Science Conference Travel Grant**
An important part of the research process is the presentation of one’s work at scholarly conferences. The purpose of this program is to provide additional funds to enable graduate students in the Faculty of Arts and Science to travel to conferences where they will present their work.

*Value:* varies to a maximum of $1,000

*Eligibility:* no citizenship restrictions; award holders must be doctoral students in the funded cohort. Graduate students may hold only one Travel Grant during their time in the department.

*Eligible Expenses:* Conference registration and abstract submission costs, travel and living expenses.

*Application:* Applications available from the Math Graduate Office; deadline October 1.

*Selection Criteria:* Past academic performance, need to attend conference for professional development, quality of abstract. Preference will be given to students near the end of their degrees.

**School of Graduate Studies Conference Grant**
The School of Graduate Studies Conference Grant program is intended to encourage students in doctoral-stream programs to actively present their research at a regional, national or international conference or equivalent academic event early in their program.

*Value:* The value will be based on the event’s registration-fee (at the student-rate) as well as a pre-determined amount based on the location of the event.

*Eligibility:* no citizenship restrictions; award holders must be doctoral-stream students; registered full-time at the time of application and at the time of conference attendance; in good standing; an active participant. Applicants need not have received confirmation of their participation in the conference at the time of application.

*Application:* For more information and deadlines: [https://www.sgs.utoronto.ca/awards/sgs-conference-grant/](https://www.sgs.utoronto.ca/awards/sgs-conference-grant/)

*Selection Criteria:* Preference will be given to applicants who are in the early stage of their academic program, have not previously attended a conference during their current program and/or who have not previously received an SGS Conference Grant. The SGS Conference Grant is not intended to be the principal source of funding. Applicants are expected to seek funding from other sources, and must list other resources that will or may be used to support their conference attendance.

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### 8. OTHER INFORMATION

The Department of Mathematics is located in the heart of the University of Toronto, which in turn is located in the heart of downtown Toronto. Students therefore have access to a wide range of facilities and services. A list appears below.
Facilities and Services

Library Facilities

The University of Toronto Library system is the 4th largest academic research library in North America. It contains over 4 million print volumes as well as a vast assortment of electronic resources. The Mathematical Sciences Library (MSL) is in the same building as the Mathematics Department. The majority of mathematics journals held by the University of Toronto are housed in the MSL, with some being held in the Gerstein Science Information Centre. The MSL’s collection also contains over 20,000 books. The MSL website includes an interface with the UofT catalogue that includes the collections of all the libraries on the St. George, Mississauga, and Scarborough campuses and includes links to online books, journals and abstract and indexing databases including MathSciNet. The MSL offers a wireless environment with study spaces for research. Each graduate student in mathematics receives a photocopying allowance. The Gerstein Science Information Centre also has a comprehensive collection of mathematics books up to 1998.

St. George T-Card Office
Koeffler Student Service Centre
214 College Street, Room 12
(Tel) 416-946-8047
tcard.office@utoronto.ca
http://tcard.utoronto.ca/

• The University of Toronto TCard is a photo ID smartcard which provides identification for academic purposes, student activities and services, facility access, and a Library Card.

Mathematics Library
40 St. George Street, Room 6141
Toronto, Ontario M5S 3G3
(Tel) 416-978-8624 | (Fax) 416-978-4107
mathlib@math.toronto.edu
http://www.library.utoronto.ca/math/

• Hours: M-F, 9 – 5; Summer hours: M-F, 9 – 4:30

Gerstein Science Information Centre
7 - 9 King’s College Circle
Toronto, Ontario M5S 1A5
(Tel) 416-978-2280 | (Fax) 416-971-2848
ask.gerstein@utoronto.ca
http://www.library.utoronto.ca/gerstein

Computer Facilities

All faculty and graduate students can request accounts on the main departmental server and the departmental compute server. The main server, coxeter, is a six-core IBM x3630 M3 server with 32GB of RAM, and the compute server, sphere, is a twelve-core IBM x3630 M3 server with 64GB of RAM, both running Red Hat Enterprise Linux 6. These accounts give access to electronic mail facilities which are also remotely accessible via encrypted IMAP/Submission or webmail, to the internet including the ability to put a webpage on the departmental webserver, to many mathematical software packages (for example, Matlab, Mathematica, pari/gp, octave), to scientific and other
graphics programs (most of the symbolic manipulators, gimp), to software compilers (supporting, for example, fortran77, fortran95, C, C++, java), to a rich mathematical software library, to mathematical typesetting programs (TeX, LaTeX), etc.

See [http://www.math.toronto.edu/intro.html](http://www.math.toronto.edu/intro.html) for a very brief introduction to the computing resources in the department. There is a public Computer Room (BA6200) in addition to machines in individual offices. University-managed wireless connectivity (the SSID is UofT) is available for most people in the department; you will need your UTORid for this. Please see [http://www.math.utoronto.ca/cms/wireless-networking/](http://www.math.utoronto.ca/cms/wireless-networking/) for more information.

Application forms for computer accounts on our systems are available from the Departmental Office (BA6290) and the Mathematics Graduate Office.

**Housing**

The university operates five graduate student residences-apartment complexes on or near the campus, ranging from unfurnished family apartments to the more conventional bed-and-board residences. In addition, the University Housing Service provides a listing of privately owned rooms, apartments and houses available for students to rent.

Students should keep in mind that accommodation could be expensive and limited, particularly in downtown Toronto. It is therefore advisable to make inquiries well in advance and to arrive in Toronto a few days prior to the start of term. Students can expect to pay anywhere between $500 to $1,000 per month on accommodation and from $300 to $500 per month on food, travel and household necessities.

**University Housing Service**

214 College Street, 1st Floor
Toronto, Ontario M5T 2Z9
(Tel) 416-978-8045 | (Fax) 416-978-1616
housing.services@utoronto.ca | [http://www.housing.utoronto.ca](http://www.housing.utoronto.ca)

**Health Services**

The University of Toronto Health Service offers medical services and referrals to private physicians for University of Toronto students. Most of these services are free of charge if you are covered under Ontario Health Coverage (OHIP), or the University Health Insurance Plan (UHIP) for visa students. OHIP application forms and information are available from the University Health Services. UHIP coverage for visa students is compulsory and is arranged during registration at the Centre for International Experience.

**Health & Wellness Centre**

214 College Street, 2nd Floor
Toronto, Ontario M5T 2Z9
(Tel) 416-978-8030 | (Fax) 416-978-2089
healthask@mail.studentlife.utoronto.ca | [http://www.healthservice.utoronto.ca/](http://www.healthservice.utoronto.ca/)

- Medical assistance for University of Toronto students
- Application forms for Ontario Health Coverage
**Centre for International Experience**  
Cumberland House, 33 St. George Street  
(Tel) 416-978-2564 | (Fax) 416-978-4090  
[http://cie.utoronto.ca/](http://cie.utoronto.ca/)

- University Health Insurance Plan (UHIP) registration at Cumberland House

**Students with Disabilities**

Services and facilities for students with disabilities are available at the University of Toronto. The University of Toronto’s Accessibility Services facilitates the inclusion of students with hidden or obvious disabilities and health conditions into university life. Services are provided to students with a documented disability, be it physical, sensory, a learning disability or a mental health condition, temporary or long-term.

**Accessibility Services**  
455 Spadina Ave., 4th Floor, Suite 400 (Just north of College Street) Toronto, Ontario, M5S 2G8  
Voice: 416-978-8060  
Fax: 416-978-8246  
TTY: 416-978-1902  
Email: accessibility.services@utoronto.ca  
[http://www.accessibility.utoronto.ca](http://www.accessibility.utoronto.ca)

**International Students**

The Centre for International Experience (CIE) offers many services to international students, including an orientation program in late August – early September, individual counselling whenever appropriate, and an English language program. In addition, the CIE contacts all foreign students once they have been accepted into the graduate program, to provide information and advice concerning immigration procedures (visa and student authorization forms), employment restrictions and authorization while in Canada, and other relevant matters.

**Centre for International Experience**  
Cumberland House, 33 St. George Street, and Koffler Student Centre, Room 201, 214 College Street  
(Tel) 416-978-2564 | (Fax) 416-978-4090 | [http://cie.utoronto.ca/](http://cie.utoronto.ca/)

- Serves international students coming to U of T and domestic students looking to go abroad  
- University Health Insurance Plan (UHIP) registration at Cumberland House

**Athletics & Recreation**

A wide range of athletic facilities are available within the university, including an arena and stadium, playing fields, swimming pools, squash, tennis, badminton, volleyball and basketball courts, running tracks, archery and golf ranges, fencing salons, exercise and wrestling rooms, dance studios, saunas, lockers and a sports store. Instruction courses, exercise classes and fitness testing are regularly offered, and there is an extensive intramural program with several levels of competition in more than 30 sports. Other recreational activities and facilities are also available within the university, such as theatre, music, pubs, dances, art exhibitions, a wide range of clubs, debates lectures and seminars, reading
rooms, cafeterias and chapels. University of Toronto students also enjoy easy access (walking distance or only a few minutes by subway) to symphony concerts, theatres, ballet, operas, movies, restaurants and shopping.

**The Athletic Centre**
55 Harbord Street
Toronto, Ontario M5S 2W6
(Tel) 416-978-3437 | (Fax) 416-978-6978 | [www.athletics.utoronto.ca](http://www.athletics.utoronto.ca)
- Multi-use health and fitness facility
- Members of the Athletic Centre also enjoy access to the state-of-the-art facilities at the new [Varsity Centre](http://www.varsitycentre.com), located at 299 Bloor Street West.
- All U of T students are automatically members of the Athletic Centre and Varsity Centre.

**Hart House**
7 Hart House Circle
Toronto, Ontario M5S 3H3
416-978-2452 | [inquiries@harthouse.ca](mailto:inquiries@harthouse.ca) | [http://www.harthouse.ca](http://www.harthouse.ca)
- University of Toronto centre for arts, culture and recreation

**Graduate Student Associations**

Every graduate student at the University of Toronto is automatically a member of the Graduate Student Union (GSU). Graduate students in the Department of Mathematics are also members of the Mathematical Graduate Students Association (MGSA). Between them, these associations sponsor many events every year, including parties, pubs, dances, outings and more serious endeavours such as seminars and lectures.

**Mathematics Graduate Student Union**
Department of Mathematics
40 St. George Street, Room 6290
Toronto, Ontario M5S 2E4
[mgsa@math.utoronto.ca](mailto:mgsa@math.utoronto.ca) | [http://www.math.toronto.edu/mgsa/](http://www.math.toronto.edu/mgsa/)

**Graduate Students’ Union**
16 Bancroft Avenue
Toronto, Ontario M5S 1C1
416-978-2391 | 416-946-8699 | [info@utgsu.ca](mailto:info@utgsu.ca) | [http://www.utgsu.ca](http://www.utgsu.ca)
- The Graduate Students’ Union at the University of Toronto represents over 18,500 students studying in over 80 departments. It advocates for increased student representation, funding, and provided services such as health insurance, confidential advice, and a voice for the graduate student body on the various committees of the University.
- Health and Dental Insurance Office: 416-978-8465 | [health@utgsu.ca](mailto:health@utgsu.ca)
Other Contacts and Sources of Information

Mathematics Graduate Office
Department of Mathematics
40 St. George St., Room 6166
Toronto, Ontario M5S 2E4
(Tel) 416-978-7894 | (Fax) 416-978-4107
gradinfo@math.toronto.edu | http://www.math.utoronto.ca/cms/graduate-program/
- All matters relating to graduate studies in mathematics at the University of Toronto
- Office Hours: M-F, 9-4

School of Graduate Studies
University of Toronto
63 St. George Street
Toronto, Ontario M5S 2Z9
(Tel) 416-978-5369 | (Fax) 416-978-4367
graduate.information@utoronto.ca | http://www.sgs.utoronto.ca
- General information concerning graduate studies at the University of Toronto
- Services include confirmation of registration letters, confirmation of degree letters, legal status changes, legal name changes
- Office Hours: M-F, 10-4

Fees Department
Office of the Comptroller
University of Toronto
215 Huron Street, 3rd Floor
Toronto, Ontario M5S 1A1
(Tel) 416-978-2142 | (Fax) 416-978-2610 | fees@finance.utoronto.ca | www.fees.utoronto.ca
- Enquiries concerning fees; payment of fees

Sexual Harassment Office
University of Toronto
215 Huron Street, 6th floor, Suite 603
416-978-3908 | http://sho.utoronto.ca/
- Students are covered by the Sexual Harassment Policy while on university premises or carrying on a university-related activity. Complaints and requests for information are confidential.

Human Resources Development Canada (HRDC)
25 St. Clair Avenue East, 1st Floor or City Hall, 100 Queen Street West, 1st Floor
1-800-206-7218
- To obtain a Social Insurance Number (in person only). Office hours: Monday-Friday, 08:30-16:00
  Supporting documentation must be original, e.g. student authorization and an offer of employment letter
- Takes an average of 4 weeks to process
Note: These are meant to be exam syllabi, not course outlines. As such, topics are not necessarily ordered as in a logical development.

The new single comprehensive exam has 12 problems (6 problems each day and 4.5 hours time limit for each day). The passing grade is 6/12.

**Algebra**

Group theory (isomorphism theorems, group actions, Sylow theorems, simple groups, direct and semidirect products, finitely generated abelian groups, solvable groups; examples: symmetric and alternating groups, dihedral groups, general and special linear groups)

Ring theory (ideals, homomorphisms, and quotient rings; for commutative rings: prime and maximal ideals, field of fractions, Euclidean domains/PID/UFD, polynomials rings, Gauss lemma, Eisenstein's criterion)

Module theory (tensor products, finitely generated modules over a PID, rational canonical form of linear transformations)

Galois theory (finite/separable/normal/Galois field extensions, tower law, splitting fields, Galois correspondence for finite Galois extensions, solvability in radicals, finite fields.)

Representations of finite groups (Schur's lemma, characters, orthogonality relations, tensor/dual representations)

Hilbert’s Nullstellensatz and applications to affine algebraic sets

**References:** Lang, Algebra
Grillet, Abstract Algebra
Dummit-Foote, Abstract Algebra

**Complex Analysis**

Elementary properties of holomorphic functions (Cauchy’s integral formula, Taylor and Laurent series, residue calculus)

Harmonic functions (Poisson integral formula and Dirichlet’s problem; maximum principle)

Conformal mapping, Riemann mapping theorem

Analytic continuation, monodromy theorem, little Picard theorem

**References:** L. Ahlfors, Complex Analysis, Third Edition, Chapters 1-4, 5.1, 5.5, 6.1, 6.2, 6.3.
W. Rudin, Real and Complex Analysis, Second Edition, Chapter 16 (except 16.4-16.7)

**Real Analysis**

Measure theory (measure spaces, Borel measures, convergence theorems such as Fatou’s lemma, monotone and dominated convergence, Fubini’s theorem, Carathéodory’s extension theorem, Radon-Nikodym)

Measure and integration on $\mathbb{R}^n$ (Lebesgue measure, change of variables formula, polar coordinates and spherical integrals, Lebesgue differentiation)

Functional analysis (Hilbert, Banach and topological vector spaces, bounded linear operators, Hahn-Banach theorem, open mapping theorem, closed graph theorem, uniform boundedness principle, weak and weak$^*$-topology, Alaoglu’s theorem, compact operators)

Hilbert spaces (orthogonal projections, orthonormal bases, Bessel’s inequality and Parseval’s identity, adjoints, spectral theory of self-adjoint operators)

Lp spaces (Hölder/Young/Minkowski inequalities, Riesz representation theorem, and interpolation)
Fourier analysis (Fourier series and transforms, Fourier inversion, Plancherel formula, $L^2$-theory and Schwartz space)

Distributions (approximation by smooth functions, weak derivatives)

References: “Real Analysis” by G. Folland;
“Real Analysis” by Stein and Shakarchi (Princeton Lectures in Analysis, Vol. III)

A working knowledge of undergraduate analysis is expected (e.g. integration and differentiation in $\mathbb{R}^n$, Implicit and inverse function theorems, uniform convergence, Weierstrass approximation theorem, Arzela-Ascoli compactness theorem, convergence of series).

**Differential Topology**
Smooth manifolds, basic examples (spheres, surfaces, tori, Lie groups, projective spaces), tangent bundles and derivatives, transversality, intersection theory, Brouwer fixed point theorem, de Rham cohomology, integration on manifolds, Stokes' theorem.

Reference: “Differential Topology” by Guillemin and Pollack

**Algebraic Topology**
Covering spaces, fundamental groups, homology, Eilenberg-Steenrod axioms, Mayer-Vietoris theorem, CW complexes, cellular homology, cohomology, cup product, universal coefficient theorem, Kunneth theorem.

Reference: “Algebraic Topology” by Hatcher.

**Partial Differential Equations**
The PDE portion of the comprehensive exam will test your understanding of the following topics:

General concepts (linear and nonlinear equations; initial and boundary conditions; existence, uniqueness, stability; classical and weak solutions; basic theory of distributions, the Sobolev spaces $H^s$ and $W^{k,p}$, Sobolev inequalities, compact embeddings)
First-order PDEs (method of characteristics. Scalar conservation laws, shocks)
Harmonic functions (mean value property, maximum principle, smoothness, Dirichlet’s principle)
Poisson equation (Dirichlet and Neumann boundary conditions, Green’s functions.
Heat equation (heat kernel, parabolic maximum principle, energy methods, smoothing properties)
Wave equation (classical solutions in dimension 1, 2, and 3. Conservation of energy, causality)

Reference: “Partial Differential Equations”, by L. Craig Evans (Chapters 1, 2, 3, 5)
A working knowledge of real analysis, multivariable calculus and ODEs is expected

**Linear Algebra and Optimization (Applied Math)**
Matrix factorizations (LU, Cholesky, QR, SVD, etc.); spectral theorems in Euclidean and Hermitian spaces
Linear optimization (e.g. simplex algorithm, Farkas lemma, primal-dual algorithm)
Basic convex optimization (Karush-Kuhn-Tucker conditions, gradient methods, and subgradients/subdifferentials).
**Probability**

Basic concepts (random variables, expectation; Kolmogorov’s extension theorem; independence and conditional expectation; Borel-Cantelli lemmas)

Laws of Large Numbers (Markov/Chebyshev/Chebyshev/Kolmogorov inequalities; weak and strong LLN; 0-1 laws, convergence of random series; stopping times)

Central Limit Theorems (weak convergence, characteristic functions; CLT for i.i.d. random variables and for triangular arrays)

References: R. Durrett, Probability: Theory and Examples Chapters 1-3; D. Panchenko, Lecture Notes (Chapters 1-3)

Familiarity with classical examples and standard discrete and continuous distributions is expected.

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**APPENDIX B: APPLIED MATH COMPREHENSIVE AND MATHEMATICAL PHYSICS EXAMINATION**

A student planning to specialize in applied mathematics or mathematical physics must pass three comprehensive exams, at least two of which are a general written exam (algebra, analysis (real and complex), topology, partial differential equations, or Probability). A third exam is set in agreement between the Graduate Coordinator and the advisor.

Other exams may be offered upon request.

**APPENDIX C: PH.D. DEGREES CONFERRED FROM 2000-2021**

**2000**

CALIN, Ovidiu (Differential Geometry) The Missing Direction and Differential Geometry on Heisenberg Manifolds
DERANGO, Alessandro (C*-Algebras) On C*-Algebras Associated with Homeomorphisms of the Unit Circle
HIRSCHORN, James (Set Theory) Cohen and Random Reals
MADORE, Blair (Ergodic Theory) Rank One Group Actions with Simple Mixing \( \mathbb{Z} \) Subactions
MARTINEZ-AVENDANO, Rubén (Operator Theory) Hankel Operators and Generalizations
MERKLI, Marco (Mathematical Physics) Positive Commutator Method in Non-Equilibrium Statistical Mechanics
MIGHTON, John (Knot Theory) Topics in Ramsey Theory of Sets of Real Numbers
MOORE, Justin (Set Theory) Topics in Ramsey Theory of Sets of Real Numbers
RAZAK, Shaloub (C*-Algebras) Classification of Simple Stably Projectionless C*-Algebras
SCOTT, Jonathan (Algebraic Topology) Algebraic Structure in Loop Space Homology
ZHAN, Yi (PDE) Viscosity Solution Theory of Nonlinear Degenerate

**2001**

COLEMAN, James (Nonlinear PDE’s) Blowup Phenomena for the Vector Nonlinear Schrödinger Equation
IZADI, Farz-Ali (Differential Geometry) Rectification of Circles, Spheres, and Classical
Geometries
KERR, David (C*-Algebras) Pressure for Automorphisms of Exact C*-Algebras and a Non-Commutative Variational Principle

OLIWA, Chris (Mathematical Physics) Some Mathematical Problems in Inhomogeneous Cosmology

PIVATO, Marcus (Mathematical Finance) Analytical Methods for Multivariate Stable Probability Distributions

POON, Edward (Operator Theory) Frames of Orthogonal Projections

SAUNDERS, David (Mathematical Finance) Mathematical Problems in the Theory of Incomplete Markets

SOLTYS-KULINICZ, Michael (Complexity) The Complexity of Derivations of Matrix Identities

VASILIJEVIC, Branislav (Mathematical Physics) Mathematical Theory of Tunneling at Positive Temperatures

YUEN, Waikong (Probability) Application of Geometric Bounds to Convergence Rates of Markov Chains and Markov Processes on $\mathbb{R}^n$

2002

HERNANDEZ-PEREZ, Nicholas (Math. Finance) Applications of Descriptive Measures in Risk Management

KAVEH, Kiumars (Algebraic Geometry) Morse Theory and Euler Characteristic of Sections of Spherical Varieties

MOHAMMADALIKANI, Ramin (Symplectic Geometry) Cohomology Ring of Symplectic Reductions

SOPROUNOV, Ivan (Algebraic Geometry) Parshin’s Symbols and Residues, and Newton Polyhedra

SOPROUNOVA, Eugenia (Algebraic Geometry) Zeros of Systems of Exponential Sums and Trigonometric Polynomials

TOMS, Andrew (Operator Algebras) On Strongly Perforated $K_0$ Groups of Simple C*-Algebras

VUKSANOVIC, Vojkan (Set Theory) Canonical Equivalence Relations

ZIMMERMAN, Jason (Control Theory) The Rolling Stone Problem

2003

ADAMUS, Janus (Analytic Geometry) Vertical components in fibre powers of analytic mappings

BUBENIK, Peter (Algebraic Topology) Cell attachments and the homology of loop spaces and differential graded algebras

HO, Nan-Kuo (Symplectic Geometry) The moduli space of gauge equivalence classes of flat connections over a compact nonorientable surface

JONG, Peter (Ergodic Theory) On the Isomorphism Problem of $p$-Endomorphisms

PEREIRA, Rajesh (Operator Theory) Trace Vectors in Matrix Analysis

STAUBACH, Wolfgang (PDE) Path Integrals, Microlocal Analysis and the Fundamental Solution for Hörmander Laplacians

THERIAULT, Nicolas (Algebraic Number Theory) The discrete logarithm problem in the Jacobian of algebraic curves

TING, Fridolin (Mathematical Physics) Pinning of magnetic vortices by external potential

TSANG, Kin Wai (Operator Algebras) A Classification of Certain Simple Stably Projectionless C*-Algebras

2004

AHMAD, Najma (Applied Math) The geometry of shape recognition via the Monge-Kantorovich optimal transportation problem (in conjunction with Brown University)

BRANKER, Maritza (Several Complex Variables) Weighted approximation in $\mathbb{R}^n$

CHEN, Oliver (Mathematical Finance) Credit barrier models

ESCOBAR AÑEL, Marcos (Mathematical Finance) Mathematical treatment of commodity...
markets
HUNG, Ching-Nam (Operator Algebras) The numerical range and the core of Hilbert-space operators
IVANESCU, Cristian (Operator Algebras) On the classification of simple C*-algebras which are inductive limits of continuous-trace C*-algebras with spectrum the closed interval [0,1]
KIRITCHENKO, Valentina (Analytic Geometry) A Gauss-Bonnet Theorem, Chern Classes and an Adjunction Formula for Reductive Groups
KUZNETSOV, Alexey (Mathematical Finance) Solvable Markov processes
LAWI, Stephan (Mathematical Finance) Exactly solvable stochastic integrals and q-deformed processes
SAVU, Anamaria (Probability) Hydrodynamic scaling limit of the continuum solid on solid model
SHAHBAZI, Zohreh (Differential Geometry) Differential Geometry of Relative Gerbes
SONG, Joon-Hyeok (Symplectic Geometry) Intersection Numbers in q-Hamiltonian Spaces
TIMORIN, Vladlen (Analytic Geometry) Rectifiable Pencils of Conics

2005
DE LOS SANTOS, Alejandro (Mathematical Finance) Liquidity risk estimation: non-gaussian AR models and quantile expansions
HAMILTON, Mark (Symplectic Geometry) Singular Bohr-Sommerfeld Leaves and Geometric Quantization
NIU, Zhuang (Operator Algebras) A classification of the tracially approximately sub-homogeneous C*-algebras
PATANKAR, Vijay (Number Theory) Splitting of Abelian Varieties
POLLANEN, Marco (Probability) Low discrepancy sequences in probability spaces

2006
CHAN, Jackson (Harmonic Analysis) Methods of variations of potential of quasi-periodic Schroedinger equation
DEJAK, Steven (Nonlinear PDE) Long-time dynamics of KdV solitary waves over a variable bottom
DOUGLAS, Andrew (Representation Theory) A classification of the finite dimensional indecomposable representations of the Euclidean algebra \( \mathfrak{e}(2) \) having two generators
FU, Guangyu (Probability) Random walks and random polynomials
HERNANDEZ CORTES, Janko (Mathematical Finance) Ergodic properties of some hidden Markov models with applications to mathematical finance
HO, Toan Minh (Operator Algebras) On the inductive limits of homogeneous algebras with diagonal morphisms between building blocks
KNAFO, Emmanuel (Number Theory) Variance of distribution of almost primes in arithmetic progressions
ROBERT GONZALEZ, Leonel (Operator Algebras) Classification of nonsimple approximate interval C*-algebras: the triangular case

2007
CALLAGHAN, Joe (Several Complex Variables) A Green’s function for \( \theta \)-incomplete polynomials
COWARD, Kristofer (Operator Algebras) The Cuntz semigroup as a classification functor for C*-algebras
LANGRIDGE, Allan (Number Theory) Values of Artin L-functions at \( s=1 \)
NAOT, Gad (Knot Theory) The Universal sl-2 link homology theory
ZHOU, Gang (Mathematical Physics) Asymptotic dynamics of trapped solitons of nonlinear
Schroedinger equations with external potentials

2008
BAIRD, Thomas (Symplectic Geometry and Algebraic Topology) Moduli spaces of flat G-bundles over nonorientable surfaces
BROOKE, David (Representation Theory) Resolving Multiplicities in the Tensor Product of Irreducible Representations of Semisimple Lie Algebras
CIUPERCA, Alin (Operator Algebras) Some Properties of the Cuntz Semigroup and an Isomorphism Theorem for a Certain Class of Non-simple C*-algebras
DONIN, Dmitry (Representation Theory and Differential Geometry) Lie Algebras of Differential Operators and D- Modules
FUCHS, Shay (Geometric Quantization) Spin^c quantization, prequantization and cutting
GERACI, Joseph (Quantum Information and Statistical Physics) On the Relation between Quantum Computation and Classical Statistical Mechanics
KLEIN, David (Symplectic geometry) Goldman Flows on Moduli Spaces of Flat Connections on Surfaces
KLEPER, Dvir (Operator Theory) Invariant Subspaces of Composition Operators on Weighted Hardy-Hilbert Spaces
LYNCH, Geoffrey (Algebraic Geometry) The Local Monodromy Operator as an Algebraic Cycle
QUINTANILLA, Maria Teresa (Mathematical Finance) Asymptotic Optimization of Risk Measures
SANTIAGO MORENO, Luis (Operator Algebras) Classification of Non-simple C*-algebras: Inductive Limits of Splitting Interval Algebras
SYLVESTRE, Jeremy (Representation Theory) Twisted Characters of Depth-zero Supercuspidal Representations of GL(n)
TIPU, Vicentiu (Number Theory) Polynomial Divisor Problems

2009
FITZPATRICK, Daniel Sean (Symplectic Geometry) Almost CR quantization via the Index of Transversally Elliptic Dirac Operators
HAMMERLINDL, Andrew (Dynamical Systems) Leaf Conjugacies on the Torus
HOVINEN, Bradford (Commutative Algebra) Matrix Factorizations of the Classical Discriminant
KISSOUNKO, Veniamine (Algebraic Geometry) The Converse of Abel’s Theorem
KREPSKI, Derek (Symplectic Geometry) Pre-quantization of the Moduli Space of Flat G-bundles
LEE, Brian C. (Symplectic Geometry) Geometric Structures on Spaces of Weighted Submanifolds
LEE, Paul Woon Yin (Symplectic Geometry and Dynamical Systems) Symplectic and Subriemannian Geometry of Optimal Transport
LI, Chao (Automorphic Forms and Representation Theory) A Local Twisted Trace Formula and Twisted Orthogonality Relations
MALONEY, Gregory (Operator Algebras) Dimension Groups and C*-algebras Associated to Multidimensional Continued Fractions
MESARIC, Jeffrey (Partial Differential Equations) Existence of Critical Points for the Ginzburg-Landau Functional on Riemannian Manifolds
MORTARI DE LACERDA, Fernando (Operator Algebras) Tracial State Space of Higher Stable Rank Simple C*-algebras
SIGLOCH, Georg (Mathematical Finance) Utility Indifference Pricing of Credit Instruments
WESSLEN, Maria (Representation Theory) A Diagrammatic Description of Tensor Product Decompositions for SU(3)
ZHURAVLEV, Vladimir (Ergodic Theory) Two Theorems of Dye in the Almost Continuous Category
ZOU, Xiangqun (Partial Differential Equations) On Blow-up of One-dimensional Heat Equations with Polynomial Nonlinearities

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**2010**
ARCHIBALD, Jana (Knot Theory) The Multivariable Alexander Polynomial on Tangles
ARIAN, Hamidreza (Mathematical Finance) Financial Engineering of the Stochastic Correlation in Credit Risk Models,
CLARK, Trevor (Dynamical Systems) Real and Complex Dynamics of Unicritical Maps
FIRSOVA, Tanya (Dynamical Systems) Dynamical Foliations
KONG, Wenbin (Nonlinear PDEs) Singularity Formation in Nonlinear Heat and Mean Curvature Flow Equations
LEUNG, Louis (Knot Theory) Classical Lie Algebra Weight Systems of Arrow Diagrams
MAZIN, Mikhail (Algebraic Geometry) Geometric Theory of Parshin Residues
MCLELLAN, Brendan (Differential Geometry) Non-Abelian Localization and U(1) Chern-Simons Theory
MONDAL, Pinaki (Algebraic Geometry) Towards a Bezout-type Theory of Affine Varieties
MORFIN RAMIREZ, Mario (Dynamical Systems) Grassmann Dynamics
SHORSER, Lindsey (Representation Theory) Scalar and Vector Coherent State Representations of Compact and Non-Compact Symplectic Groups in a Unitary Basis
SOKIC, Miodrag (Set Theory) Ramsey Property of Posets and Related Structures
TZANETEAS, Tim (Mathematical Physics) Abrikosov Lattice Solutions of the Ginzburg-Landau Equations of Superconductivity
ZHANG, Yichao (Analytic Number Theory) L-functions in Number Theory
ZOGHI, Masrour (Symplectic Geometry) The Gromov Width of Coadjoint Orbits of Compact Lie Groups
ZWIERS, Ian (Nonlinear PDEs) Standing Ring Blowup Solutions for the Cubic Nonlinear Schroedinger Equation

**2011**
ANAPOLITANOS, Ioannis (Math Physics) On van der Waals forces
BLOEMENDAL, Alexander (Probability) Finite Rank Perturbations of Random Matrices and Their Continuum Limits
CARRASCO, Pablo (Dynamical Systems) Compact Dynamical Foliations
DANCSO, Zsuzsanna (Knot Theory) A Universal Finite Type Invariant Of Knotted Trivalent Graphs
HOEHN, Logan (Set-theoretic topology) Non-Chainable Continua and Lelek's Problem
JASINSKI, Jakub (Combinatorics) Hrushovski and Ramsey Properties of Classies of Finite Inner Product Structures, Finite Euclidean Metric Spaces and Boron Trees
LAI, Chung Lun Alan (Noncommutative Geometry) On the JLO Character and Loop Quantum Gravity
MARTINEZ RANERO, Carlos (Set Theory) Contributions towards a Fine Structure Theory of Aronszajn Orderings
PASS, Brendan (Geometric Analysis) Structural Results on Optimal Transportation Plans
PIGOTT, Brian (Partial differential equation) Low Regularity Stability for Subcritical Generalized Korteweg-de Vries Equations
ROWE, Barry (Operator Theory) The Left Regular Representation of a Semigroup
SHARTSER, Leonid (Geometry and Topology) De Rham Theory and Semialgebraic Geometry
SQUIRES, Travis (Algebra) Lie 2-Algebras as Homotopy Algebras Over a Quadratic Operad
TIKUISIS, Aaron (Operator Algebra) The Cuntz Semigroup of C(X,A)
UREN, James (Differential geometry) Toric Varieties Associated with Moduli Spaces
VODA, Mircea (Several Complex Variables) Loewner Theory in Several Complex Variables and Related Problems

**2012**
BAILEY, Michael (Differential Geometry) On the local and global classification of generalized complex structures
BURDA, Yuri (Algebraic Geometry) Topological Methods in Galois Theory
CHO, Peter Jaehyun (Number Theory) L-functions and Number Theory
CHU, Karene (Geometric Topology) Flat Virtual Pure Tangles
DUDKO, Artem (Dynamical Systems) Dynamics of holomorphic maps: Resurgence of Fatou coordinates, and Poly-time computability of Julia sets
FONTAINE, Bruce (Representation Theory) Bases for Invariant Spaces and Geometric Representation Theory
FRANCETIC, Nevena (Discrete Math) Covering Arrays with Row Limit
ISGUR, Abraham (Combinatorics) Solving Nested Recursions With Trees
KAMALINEJAD, Ehsan (Analysis of PDE) Optimal Transport Approach to Non-linear Evolution Equations
KINZEBULATOV, Damir (Several Complex Variables) Geometric analysis on solutions of some differential inequalities and within restricted classes of holomorphic functions
LEE, Stephen Peter (Algebra) The Pure Virtual Braid Group is Quadratic
LI-BLAND, David (Lie Theory) Courant Algebroids and their Applications
MAZZEO, Elio (Ergodic Theory) On C^1-rigidity for maps with a break point
PARSONS, Todd (Mathematical Biology) Asymptotic Analysis of Some Stochastic Models from Population Dynamics and Population Genetics
PETZKA, Henning (C*-Algebras) Stably non-stable C*-algebras with no bounded trace
RICHARDS, Geordie (Probability and PDE) Maximal-in-time behavior of deterministic and stochastic dispersive partial differential equations
SANKARAN, Siddarth (Arithmetic Geometry) Special cycles on Shimura curves and the Shimura lift
SHAHROKHI TEHRANI, Shervin (Arithmetic Geometry) Non-holomorphic cuspidal automorphic forms of GSp(4;𝔸) and the Hodge structure of Siegel threefolds
TAM, Kam-Fai (Number Theory) Transfer relations in essentially tame local Langlands correspondence
VERA PACHECO, Franklin (Algebraic Geometry) Resolution of singularities of pairs preserving semi-simple normal crossings
WATTS, Jordan (Differential Topology) Diffeologies, Differential Spaces, and Symplectic Geometry
YANG, Jihyeon Jessie (Algebraic Geometry) Tropical Severi Varieties and Applications

2013
AMIR-KHOSRAVI, Zavosh (Arithmetic Geometry) Moduli of Abelian Schemes and Serre’s Tensor Construction
ANGHEL, Catalina (Number Theory) The self-power map and its image modulo a prime
BARTOSOVA, Dana (Set Theory) Topological dynamics in the language of near ultrafilters and automorphism groups of ω-homogeneous structures
CHEN, Shbing (Analysis of PDE) Convex solutions to the power-of-mean curvature flow, conformally invariant inequalities and regularity results in some applications of optimal transportation
DOTTERRER, Dominic (Metric Geometry) The (co)isoperimetric problem in (random) polyhedra
LAPTYEVA, Nataliya (Number Theory) A Variant of Lehmer’s Conjecture in the CM Case
LI, Travis (Differential Geometry) Constructions of Lie Groupoids
LIU, Xiao (Nonlinear PDEs) Analytical and numerical results for some classes of nonlinear Schrödinger equations
MOURTADA, Mariam (Number Theory) The distribution of values of logarithmic derivatives of real L-functions
PYM, Brent (Algebraic Geometry) Poisson structures and Lie algebroids in complex geometry
SMITH, Kathleen (Symplectic Geometry and Topology) Connectivity and Convexity Properties of the Momentum Map for Group Actions on Hilbert Manifolds
WALLS, Patrick (Number Theory) The Theta Correspondence and Periods of Automorphic Forms

2014
BRODSKY, Ari (Set Theory) A Theory of Stationary Trees and the Balanced Baumgartner-Hajnal-Todorcevic Theorem for Trees
BURKO, Robert (Number Theory) Computing the Zeta Function of Two Classes of Singular Curves
CAVIEDES CASTRO, Alexander (Symplectic Geometry) Upper bounds for the Gromov width of coadjoint orbits of compact Lie groups
CHAMBERS, Gregory (Metric and Riemannian Geometry) Optimal homotopies of curves on surfaces
DONNELLY, Ryan (Stochastic Control and High Frequency Trading) Effects of Ambiguity Aversion on High Frequency Trading
FISHER, Jonathan (Symplectic and algebraic geometry, group actions) The Topology and Geometry of Hyperkähler Quotients
HART, Eric (Probability) Hölder Continuity of the Integrated Density of States in the One-Dimensional Anderson Model
HANNIGAN-DALEY, Bradley (Algebraic Geometry) Hypertoric varieties and wall-crossing
MAYOST, Daniel (Differential Geometry) Applications of the signed distance function to surface geometry
MORGAN, Stephen (Representation Theory) Quantum Hamiltonian reduction of W-algebras and category $\mathcal{O}$
ROBINSON, Patrick (Poisson Geometry) The Classification of Dirac Homogeneous Spaces
VENA, Lluis (Combinatorics) The removal property for linear configurations in compact abelian groups
WATSON, Nicola (Operator Algebras) On the Structure of Nuclear C*-algebras with Real Rank Zero

2015
ANGELOPOULOS, Ioannis (Partial Differential Equations) Nonlinear waves on extremal black hole spacetimes
CHOW, Aaron (Number Theory) Applications of Fourier coefficients of modular forms
CHTERENTAL, Oleg (Virtual Braids) Virtual Braids and Virtual Curve Diagrams
DAHL, Alexander (Analytic Number Theory) Subconvexity for a double Dirichlet series and non-vanishing of L-functions
EAGLE, Christopher (Set Theory) Topological Aspects of Real-Valued Logic
FENG, Jackson (Probability) Rescaled Directed Random Polymer in Random Environment in Dimension $1 + 2$
GEORGE, William (Number Theory) Lifting Problems, Cross-fiberedness, and Diffusive Properties on Elliptic Surfaces
GUDIM, Mikhail (Homological algebra) Equivariant Modules
HANSON, Brandon (Analytic Number Theory, Combinatorics) Character Sum Estimates in Finite Fields and Applications
LIOKUMOVICH, Yevgeniy (Riemannian geometry) Sweepouts of Riemannian surfaces
PAVLOV, Alexander (Homological algebra, algebraic geometry) Betti Tables of Maximal Cohen-Macaulay Modules over the Cones of Elliptic Normal Curves
PAWLIUK, Michael (Set Theory) Amenability and Unique Ergodicity of the Automorphism Groups of all Countable Homogeneous Directed Graphs

43
RAHMAN, Mustazee (Probability) Sub-optimality of local algorithms on sparse random graphs
ROWE, Daniel (Geometric Representation Theory) Lusztig Slices in the Affine Grassmannian and Nilpotent Matrices
SOUKUP, Daniel (Set Theory) Colouring problems of Erdős and Rado on infinite graphs

2016
BAZETT, Trefor (Equivariant K-theory) The equivariant K-theory of commuting 2-tuples in SU(2)
CROOKS, Peter (Lie Theory and Equivariant Geometry) The Equivariant Geometry of Nilpotent Orbits and Associated Varieties
ESKANDARI, Payman (Number theory) Algebraic Cycles, Fundamental Group of a Punctured Curve, and Applications in Arithmetic
FOURNODAVLOS, Grigorios (Geometric Analysis) Stability of singularities in geometric evolutionary PDE
GLYNN-ADLEY, Parker (Quantitative geometry) Width, Ricci Curvature, and Bisecting Surfaces
HALACHEVA, Iva (Knot Theory, Representation Theory) Alexander-type invariants of tangles, Skew Howe duality for crystals and the cactus group
HOLDEN, Tyler (Geometry) Convexity and Cohomology of the Based Loop Group
KOTOWSKI, Marcin (Probability) Random Schroedinger operators with connections to spectral properties of groups and directed polymers
KOTOWSKI, Michal (Probability) Return probabilities on groups and large deviations for permuton processes
LISHAK, Boris (Quantitative geometry) Balanced Presentations of the Trivial Group and 4-dimensional Geometry
LIVINSKYI, Ivan (Number Theory) On the integrals of the Kudla-Millson theta series
MOUSAVIDEHSHIKH, Ali (Homological algebra) Constructing endomorphism rings of large finite global dimension
STEWART, Andrew (Probability, Random Walks on Groups) On the scaling limit of the range of a random walk bridge on regular trees
THOMPSON, Kyle (PDEs) Dynamics of Superconducting Interfaces
THIBAULT, Louis-Philipppe (Representation Theory of Algebras) Applications of algebraic microlocal analysis in symplectic geometry and representation theory
VOLTZ, Jeremy (Probability Theory) Two results on Asymptotic Behaviour of Random Walks in Random Environment
WEEKES, Alexander (Representation theory) Highest weights for truncated shifted Yangians

2017
BALEHOWSKY, Tracey, (Geometric inverse problems) Recovering a Riemannian Metric from Knowledge of the Areas of Properly-Embedded, Area-Minimizing Surfaces
KLYS, Jack (Number Theory) Statistics of class groups and related topics
LANE, Jeremy (Symplectic geometry) On the topology of collective integrable systems
LOIZIDES, Yiannis (Symplectic geometry) Norm-square localization for Hamiltonian LG-spaces
LUK, Kevin (Algebraic geometry) Logarithmic algebroids and line bundles and gerbes
LUTLEY, James (Diagonal maps between RFD algebras) The Structure of Diagonally Constructed ASH Algebras
MRACEK, James (Symplectic geometry) Applications of algebraic microlocal analysis in symplectic geometry and representation theory
REISS, David (PDEs) Global Well-Posedness and Scattering Of Besov Data For the Energy-Critical Nonlinear Schrödinger Equation by
SCHACHTER, Benjamin (Optimal transportation, calculus of variations, PDEs) An Eulerian Approach to Optimal Transport with Applications to the Otto Calculus
SMITH, Jerrod (Representation theory of p-adic groups) Construction of relative discrete series representations for p-adic GLn
YANG, Jonguk (Complex dynamics and renormalization) Applications of Renormalization to Irrationally Indifferent Complex Dynamics
ZAMAN, Asif (Analytic number theory) Analytic estimates for the Chebotarev Density Theorem and their applications

2018
AMELOTTE, Steven (Algebraic Topology) Unstable Homotopy Theory Surrounding the Fiber of the $p^\text{th}$ Power Map on Loop Spaces of Spheres
BRIGGS, Benjamin (Homological algebra, representation theory) Local Commutative Algebra and Hochschild Cohomology Through the Lens of Koszul Duality
DIXIT, Anup (Number Theory) The Lindelof Class of L-Functions
ENNS, John (Number Theory) On mod p local-global compatibility for unramified GL3
FEIZMOHAMMADI, Ali (Partial Differential Equations) Unique Reconstruction of a Potential from the Dirichlet to Neumann Map in Locally CTA Geometries
FUSCA, Daniel (Geometric mechanics, infinite dimensional Hamiltonian systems) A groupoid approach to geometric mechanics
GELINAS, Vincent (Homological Algebra) Contributions to The Stable Derived Categories of Gorenstein Rings
HERNANDEZ BELLON, Julio (Financial Math) Correlation Model Risk and Non Gaussian Factor Models
FULGENCIO, Lopez (Set Theory) Construction schemes and their applications
LAREAU-DUSSAULT, Rosemonde (Optimal transport) Coupled Education and Labour Market Models
MANGEREL, Alexander (Analytic number theory) Topics in Multiplicative and Probabilistic Number Theory
NIKOLAEV, Nikita (Complex algebraic geometry and mathematical physics) Abelianisation of Logarithmic Connections
VO, Huan (Knot Theory) Alexander Invariants of Tangles via Expansions
WOLSKE, Zackary (Number Theory- monogenic fields) Number fields with Large Minimal Index
WU, Nan (Differential Geometry, massive data analysis) Differential Geometry Approach For Unsupervised Machine Learning Algorithms
ZHANG, Shuangjian (Optimal transportation and its applications) Existence, Uniqueness, concavity and geometry of the monopolist’s problem facing consumers with nonlinear price preferences

2019
BISCHOFF, Francis (Differential geometry and mathematical physics) Morita Equivalence and Generalized Kähler Geometry
CHEN, Li (Mathematical physics) Macroscopic Electrostatics at Positive Temperature from the Density Functional Theory
CHICHE-LAPIERRE, Val (Number theory, Arithmetic geometry) Length of elements in a Minkowski basis for an order in a number field (or a ring of integers of a number field)
DAUVERGNE, Duncan (Probability) Random sorting networks, the directed landscape, and random polynomials
ESENTEPE, Ozgür (Geometric and combinatorial representation theory) Annihilation of Cohomology over Gorenstein Rings
GUEVARA PARRA, Francisco Javier (Set theory and combinatorics) Analytic spaces and their Tukey types
JI, Jia (Symplectic geometry, geometric applications of quantum field theory) Volume Formula and Intersection Pairings of N-fold Reduced Products
LIU, Chia-Cheng (Representation Theory, Geometric and combinatorial representation theory) Semi-infinite Cohomology, Quantum Group Cohomology, and the Kazhdan-Lusztig Equivalence
MARTEL, Justin (Mathematical physics, mathematical economics, inequalities, optimization, partial differential equations) Applications of Optimal Transport to Algebraic Topology: A Method for Constructing Spines from Singularity
MILLER, Evan (Mathematical physics, mathematical economics, inequalities, optimization, partial differential equations) The Navier-Stokes strain equation with applications to enstrophy growth and global regularity
MONIN, Leonid (Algebra, geometry, theory of singularities) Overdetermined systems of equations, Newton Polyhedra, and Resultants
PARSCH, Fabian (Geometric calculus of variations, quantitative aspects of topology of manifolds, representation theory) Geodesic Nets with Few Boundary Points
RAJARATNAM, Krishan (Mathematical physics) Abrikosov lattice solutions of the ZHK Chern-Simons equations
ZEROUALI, Jihad (Symplectic geometry, mathematical physics, Lie theory) Twisted conjugation, quasi-Hamiltonian geometry, and Duistermaat-Heckman measures
ZHU, Zhifei (Riemannian geometry) Geometric inequalities on Riemannian manifolds

2020
CARRUTH, Nathan (Applied Mathematics) Focussed Solutions to the Einstein Vacuum Equations
DRANOWSKI, Anne (Geometric representation theory) Comparing two perfect bases
ENS, Travis (Knot theory, quantam algebra) On Braidors: An Analogue of the Theory of Drinfel’d Associates for Braids in an Annulus
GARDNER, Adam (Applied Mathematics) Instability of electroweak homogeneous vacua in strong magnetic fields
KO, Justin (Probability) The Free Energy of Spherical Vector Spin Glasses
KUNDU, Debanjana (Iwasawa theory) Iwasawa Theory of Fine Selmer Groups
MATVIICHUK, Mykola (Poisson structures) Quadratic Poisson brackets and co-Higgs fields
NAVARRO LAMEDA, Beatriz (Probability) On Global Solutions of the Parabolic Anderson Model and Directed Polymers in a Random Environment
OSWAL, Abhishek (Number theory) A non-archimedean definable Chow theorem
PHAM, Khoa (Geometric and combinatorial representation theory) Multiplication of generalized affine Grassmannian slices and comultiplication of shifted Yangians
PIKE, Jeffrey (Poisson geometry, Lie theory) Weil Algebras and Double Lie Algebroids
TALIDOU, Afroditi (Applied Mathematics) Near-pulse solutions of the FitzHugh-Nagumo equations on cylindrical surfaces
VERBERNE, Yvon (Low dimensional topology) Pseudo-Anosov homeomorphisms constructed using positive Dehn twists

2021
ALBOIU, Mihai (C*-Algebra) The Stable Rank of Diagonal Ash Algebras
KAWACH, Jamal (Set theory) Approximate Ramsey Methods in Functional Analysis
MEHTA, Arthur (Applied Mathematics) Entanglement and non-locality in games and graphs
PARK, Seong Hyun (Applied Mathematics) Adaptive myelination and its synchronous dynamics in the Kuramoto network model with state-dependent delays
RICHARDS, Larissa (Geometric function theory) Convergence rates of random discrete model curves approaching SLE curves in the scaling limit
SIAD, Artane (Arithmetic geometry, arithmetic statistics) Monogenic Fields with Odd Class Number
TAWFIK, Selim (Geometry) Fusion Product of D/G-Valued Moment Maps

APPENDIX D: THE FIELDS INSTITUTE FOR RESEARCH IN MATH SCIENCES

The Fields Institute for Research in Mathematical Sciences was created in November 1991 with major funding from the Province of Ontario, the Natural Sciences and Engineering Research Council of Canada, and McMaster University, the University of Toronto, and the University of Waterloo. In September 1996 it moved from its temporary location in Waterloo to its permanent site, a new building located at 222 College Street in Toronto, next to the University of Toronto Bookstore. In addition to the three principal sponsoring universities, about twenty universities across Canada are affiliated with it.

The mandate of the Fields Institute specifically includes the training of graduate students and this function is given a higher profile than at other similar mathematics research institutes. All major programs run at the institution contain graduate courses which students at any university affiliated with the institute may take for credit and the organizers of major programs are expected to set aside some money to make it possible for graduate students to participate in their program.

APPENDIX E: 2021-22 INSTRUCTONS FOR COURSE ENROLMENT ON ACORN

Visit this webpage for a detailed, step-by-step guide to using ACORN for course enrolment, program enrolment, checking finances, updating address and contact information, printing or ordering transcripts, and more.

Graduate students can access ACORN to:

- View, request, add, drop, or waitlist for courses
- View personal timetable
- View/Change address, telephone numbers, email, safety abroad address, and emergency contact information
- View academic history, including final grades
- Order transcripts
- View your financial account information (invoices, account details, payments)
- Defer payment of tuition (available to recipients of a funding commitment from the graduate unit and to students approved Ontario, Canada and some US government student loan)
- Update direct deposit details
- Print “Educational Credit” tax forms (T2202A)
- Order convocation tickets
- And more

Student Responsibility

While academic advisors, faculty, and staff are available to assist and advise, it is ultimately the student's responsibility to keep personal and academic information up to date at all times and to follow all University, SGS, departmental and program regulations, requirements and deadlines. ACORN makes it easier for you to check and correct this information. If questions arise about requirements, policies and procedures, you are responsible for seeking answers to these questions from staff and advisors.
University of Toronto Email
The Graduate Office and other university offices may send important information to you by email. It is your responsibility to ensure that your email address, mailing/permanent address, and telephone numbers are up to date at all times.

Under University policy, students are required to maintain a University based email account (i.e., ending in utoronto.ca or toronto.edu), record it in ACORN, and regularly check for messages. The University will send official correspondence to your utoronto email account.

UTORid and JOINid
Students use their UTORid or JOINid and password to log onto ACORN.

Every applicant to the University of Toronto is assigned a JOINid. It was provided to you through an email from SGS (admissions.sgs@utoronto.ca).

When you begin your studies at U of T, your JOINid will become your UTORid, which will allow you to access a number of services such as email, library resources, and Quercus, the University’s student portal and learning management system. Many courses use the portal to provide online materials, discussion groups, quizzes, and more.

Enabling your JOINid
You must enable your JOINid and create a password if you wish to access ACORN before arriving on campus. This can be done online – you do not have to be here in person. Your JOINid will not work until you enable and create a password for it.

If you have trouble locating the email sent to you with your JOINid, please contact admissions.sgs@utoronto.ca.

If you lose your JOINid password, you can use the enabling site to change your password. Please note that updates to passwords can take 24 hours to roll over in the system. If you are unable to reset your password using the enabling site, please call 978-HELP, or email help.desk@utoronto.ca to reset your password. Please specify that you have already enabled your JOINid.

Activating your UTORid
Instructions on how to activate your UTORid are provided to you by the TCard Office. There are online procedures in place to allow you to obtain your UTORid activation instructions and U of T email starting June 1st over a video conference call with a TCard staff member. When the University re-opens, you will be required to visit a TCard office in person to obtain your TCard and provide documentation to validate your legal status in Canada.

Declaration
The use of ACORN to enrol in courses means that you agree to abide by all of the academic and non-academic rules and regulations of the University, the School of Graduate Studies, and the graduate unit in which you are registered. It also means that you agree to assume the obligation to pay academic and incidental fees according to the policies and requirements of the University of Toronto. You normally will use ACORN to add or cancel courses. If, for extraordinary reasons, you are unable to use the system, contact your graduate unit as soon as possible.

Students are expected to be responsible when using the system and should not attempt to flood it with requests, or to automate the process of course enrolment. Such activity may clog the system so that
other students may be denied access or experience degraded performance. Any student(s) attempting such activity may be denied access to ACORN until after the relevant registration period.

Checking course status
You are responsible for knowing the status of your course requests at all times. This information can be obtained through ACORN. The following are possible statuses:

REQ: Course requested. Must be resolved/approved by the last date to add a course.
INT: Course requested pending instructor approval in addition to co-ordinator's/advisor's approval.
APP: Request approved. Student is enrolled in course.
REF: Request denied. Student is not enrolled and may not make another request for this course via the web during this session.
CAN: Course cancelled (student withdrew from course before deadline)
WAIT: No room in the meeting section. Student has been placed on a waiting list based on category and will be enrolled automatically if space becomes available.
DWAIT: Student has cancelled place on the waiting list or been removed.
Cancelling or withdrawing from courses
You may cancel or withdraw from individual courses up to certain deadlines. Before doing this, however, you are advised to consult with your advisor or departmental office.

The deadlines to drop courses without academic penalty are as follows:

- **October 25, 2021** for Fall session full or half courses;
- **February 20, 2022** for full-year and Winter session courses;
- **May 27, 2022** for May-to-June F section courses; and
- **July 18, 2022** for July-to-August S section courses.

If you miss the deadline to drop a course:

1. Complete the Add/Drop Course(s) form and submit it to your Graduate Administrator, along with a letter of rationale with supporting documentation (e.g. medical certificate).

2. The graduate unit will consider the request and, if supported, will forward the request to SGS for review.

3. If approved by SGS, the transcript notation of WDR (Withdrawn without Academic Penalty) will be assigned by SGS to the course. The WDR notation carries no credit for the course and is not considered for averaging purposes.

Some graduate units offer modular courses which have enrolment deadlines that do not conform to the deadlines above. Modular courses with non-standard start/end dates require the graduate unit to establish suitable drop dates. Please check with the graduate unit offering modular courses for the drop dates.

Dropping courses may have implications for your progress in the program. For details, check with your departmental Graduate Administrator.

Please note that withdrawing from all your courses does not constitute a withdrawal from your program. To do so, you must complete a Program Withdrawal Form. Dropping courses prior to deadlines or withdrawing from a program does not guarantee a refund. Information on fee refunds and deadlines is outlined at www.fees.utoronto.ca.

Final results
Final grades in courses can be accessed in ACORN by selecting Academic History.

Grades can be viewed after the following dates. If a grade is not available after these dates, contact your instructor or the graduate unit offering the course.

- **July 14, 2021** Summer Session (first term)
- **September 8, 2021** Summer Session (full summer and second term)
- **January 12, 2022** Fall Session
- **May 18, 2021** Winter Session (and Fall/Winter courses)

Important dates and deadlines:
Important dates and registration deadlines are available on the SGS Calendar and can be found here. Please follow the departmental deadline dates.