Instructor: B. Khesin

Graduate course MAT 1126HS
Spring 2018
“Lie Groups and Hamiltonian PDEs”

Syllabus:

I. Introduction and main notions.
   1. Lie groups and Lie algebras.
   2. Adjoint and coadjoint orbits.
   3. Central extensions.
   4. The Lie–Poisson (or Euler) equations for Lie groups.
   5. Bihamiltonian systems.

II. Geometry of infinite-dimensional Lie groups and their orbits.
   1. Affine Kac–Moody Lie algebras and groups.
      1.1. Definition of the affine Kac–Moody Lie algebras.
      1.2. Affine Lie groups.
      1.3. Their coadjoint orbits.
      1.4. The quotient (WZW) construction of the affine groups.
   2. The Virasoro algebra and group. The KdV equation.
      2.1. Definitions.
      2.2. The group of circle diffeomorphisms.
      2.3. The Virasoro coadjoint action.
      2.4. Virasoro coadjoint orbits.
      2.5. The Virasoro group and Korteweg-de Vries equation.
      2.6. Bihamiltonian structure of the KdV.
      3.1. The Lie group of volume-preserving diffeomorphisms and its Lie algebra.
      3.2. Coadjoint action and Casimirs.
      4.1. Pseudodifferential operators and cocycles on them.
      4.2. The Lie group of pseudodifferential operators of complex degree.
      4.3. Integrable KP-KdV hierarchies.

III. Applications to PDEs in geometric fluid dynamics. (time permitted)
   1. Ideal hydrodynamics and optimal mass trasport. Otto’s calculus.
   2. Compressible fluid dynamics.
   4. Structures on and dynamics of vortex sheets.
   5. $H^1$ geometry on diffeomorphism groups and Fisher-Rao metric.
References:


Prerequisites:

A basic course (or familiarity with main notions) of symplectic geometry would be helpful.