

DEPARTMENT OF MATHEMATICS
University of Toronto

Complex Analysis Exam

1.5 hours

September 9, 2015

There are three questions, all of equal value.
Show all your work.

1. Evaluate via residues

$$\int_{-\infty}^{\infty} \frac{\cos \lambda x}{x^2 + 2x + 5} dx$$

where $\lambda > 0$.

2. Does there exist a holomorphic function $f : \mathbb{C} \setminus \{0\} \rightarrow \mathbb{C}$, such that

$$|f(z)| \geq \frac{1}{\sqrt{|z|}}$$

for all $z \in \mathbb{C} \setminus \{0\}$? Either give an example or prove that no such function exists.

3. Let f be a holomorphic mapping from the unit disk to itself which is *not* the identity. Show that f has at most one fixed point.