

TAYLOR SERIES



March 25th, 2019

For next lecture

For Wednesday (Mar 27), watch the videos:

- Analytic functions: 14.7, 14.8
- New power series: 14.9, 14.10

Taylor polynomial of a polynomial

Let $f(x) = x^3$

- 1 Write the 2nd Taylor polynomial P_2 for f at 0
- 2 Write the 3rd Taylor polynomial P_3 for f at 0
- 3 Write the 2nd Taylor polynomial for f at 1.
- 4 Write the 3rd Taylor polynomial for f at 1

Let $f(x) = x^3$

- 1 Write the 2nd Taylor polynomial P_2 for f at 0
- 2 Write the 3rd Taylor polynomial P_3 for f at 0
- 3 Write the 2nd Taylor polynomial for f at 1.
- 4 Write the 3rd Taylor polynomial for f at 1

You have learned the Maclaurin series for the following functions

$$f(x) = e^x, \quad g(x) = \sin x, \quad h(x) = \cos x$$

Compute the interval of convergence of each of these three series.

Write the Taylor series...

- 1 for $f(x) = e^x$ at $a = 2$
- 2 for $g(x) = \sin x$ at $a = \frac{\pi}{4}$
- 3 for $H(x) = \frac{1}{x}$ at $a = 3$

You can do these problems in two ways:

- Method 1: Compute the first few derivatives, guess the pattern (and prove it by induction).
- Method 2: Use the substitution $u = x - a$ and reduce it to an old problem (without computing any derivative).

An interesting power series - *Homework*

Denote by a_n the n -th decimal digit of π .

Define the power series $S(x) = \sum_{n=1}^{\infty} a_n x^n$.

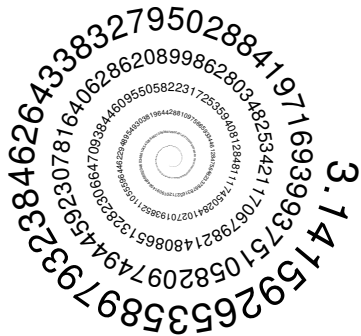
Find the radius of convergence and the interval of convergence of S .

Hint?

$\pi=3.1415926535897932384626433832795028841971693993751058209749445923078164062862089$

So $a_1 = 1$, $a_2 = 4$, $a_3 = 1$, $a_4 = 5 \dots$

Hint (to be continued)



Hint (to be continued)

3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148
0865132823066470938446095505822317253594081284811174502841027019385211055596446229489549303819644288109756
65933446128475648233786783165271201909145648566923460348861045432664821339360726024914127372458700660631558
8174881520920628292540917153643678925903600113305305446204665213841469519415116094333057270365759591953092
186117381932611793105118548074462379962749567351885752724891227938183011949129833673632406566430860213949
4639522473719070217986094370277053921717629317675238467481846766940513200056812714526356082778577134275778
9609173637178721468440901224953430146549585371050792279689258923542019956112129021960864034418159813629774
7713099605187072113499999983729780499510597317328160963185950244594553469083026425223082533446850352619311
88170101000313783875288658753320838142061717766914730359825349042875546873115956286388235378759375195778185
7780532171226806613001927876611195909216420198938095257201065485863278865936153381827968230301952035301852
9689957736225994138912497217752834791315155748572424541506959508295331168617278558890750983817546374649393
1925506040092770167113900984882401285836160356370766010471018194295559619894676783744944825537977472684710
4047534646208046684259069491293313677028989152104752162056966024058038150193511253382430035587640247496473
2639141992726042699227967823547816360093417216412199245863150302861829745557067498385054945885869269956909
2721079509302955321165344987202755960236480665499119881834797753636980742654252786255181841757467289097
77727938000816470600161452491921732172147723501414197356854816136115735255213347574184946843852332907394
1433345477624168625189835694855620992192221842725502542568876717904946016534668049886272327917860857843838
2796797668145410095388378636095068006422512520511739298489608412848862694560424196528502221066118630674427
8622039194945047123713786960956364371917287467764657573962413890865832645995813390478027590099465764078951
26946839835259570982582262052248940772671947826848260147699090264013639443745530506820349625245717493996514
3142980919065927093722169646151157098583874105978859597729754989301617539284681382686838689427415599185592
52459539594310499725224680845987273644695848653836736222626099124608051243884390451244136549762780797715691
4359977001296160894416948685558484063534220722258284886481584560285060168427394522674676788952521385225499
5466672782398645659611635488623057745649803559363456817432411251507606947945109659609402522887971089314566
9136867228748940560101503308617928680920874760917824938589009714909675985261365549781893129784821682998948
722658804857564014270477551323796414512537462343645428584447952658678210511413547357395231134271661021359
695362314429524849371871101457563040359027993440374200731057853906219838744780847848968332144571386875194350
6430218453191048481005370614680674919278191197939952061419663428754440643745123718192179998391015919561814
6751426912397489409071864942319615679452080951465502252316038819301420937621378559566389377870830390697920
7734672218256259966150142150306803844773454920260541466592520149744285073251866600213243408819071048633173
4649651453905796268561005508106658796998163574736384052571459102897064140110971206280439039759515677157700
4203378699360072305587631763594218731251471205329281918261861258673215791984148488291644706095752706957220
9175671167229109816909152801735067127485832228718352093539657251210835791513698820914442100675103346711031
412671112609096595162992150107016515116951714276576192515565098409999950992287245529221625507647918525992