MAT137Y1 – LEC0501

Calculus!

Taylor series

March 25th, 2019

For next lecture

For Wednesday (Mar 27), watch the videos:

- Analytic functions: 14.7, 14.8
- New power series: 14.9, 14.10

Taylor polynomial of a polynomial

Let $f(x) = x^3$

1. Write the 2nd Taylor polynomial P_2 for f at 0
2. Write the 3rd Taylor polynomial P_3 for f at 0
3. Write the 2nd Taylor polynomial for f at 1.
4. Write the 3rd Taylor polynomial for f at 1

Interval of convergence

You have learned the Maclaurin series for the following functions

$$f(x) = e^x, \qquad g(x) = \sin x, \qquad h(x) = \cos x$$

Compute the interval of convergence of each of these three series.

Taylor series not at 0

Write the Taylor series...

1. for $f(x) = e^x$ at $a = 2$
2. for $g(x) = \sin x$ at $a = \frac{\pi}{4}$
3. for $H(x) = \frac{1}{x}$ at $a = 3$

You can do these problems in two ways:

- Method 1: Compute the first few derivatives, guess the pattern (and prove it by induction).
- Method 2: Use the substitution $u = x - a$ and reduce it to an old problem (without computing any derivative).

An interesting power series - *Homework*

Denote by a_n the n-th decimal digit of π.

Define the power series $S(x) = \sum_{n=1}^{\infty} a_n x^n$.

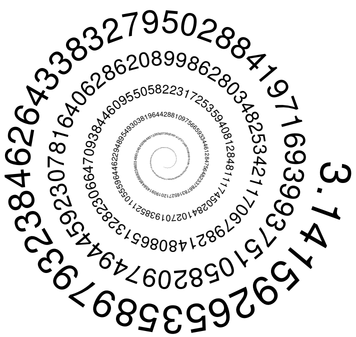
Find the radius of convergence and the interval of convergence of S.

Hint?

π=3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679

So $a_1 = 1,\ a_2 = 4,\ a_3 = 1,\ a_4 = 5 \ldots$

Hint (to be continued)



Hint (to be continued)

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482534211706798214808
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