
DEFINITION OF SERIES



UNIVERSITY OF
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For next lecture

For Wednesday (Mar 6), watch the videos:

- Properties of series: 13.5, 13.6, 13.7, 13.8, 13.9

Comment: I wrote some notes about last lecture.

<http://www.math.toronto.edu/campesat/ens/1819/lec38-notes.pdf>

Rapid questions: review of improper integrals

$$\textcircled{1} \int_1^{+\infty} \frac{1}{x^2} dx$$

$$\textcircled{2} \int_1^{+\infty} \frac{1}{\sqrt{x}} dx$$

$$\textcircled{3} \int_1^{+\infty} \frac{1}{x^2 + \sqrt{x}} dx$$

For 3, 6 and 9: draw the graphs of $y = \sqrt{x}$ and $y = x^2$.

Rapid questions: review of improper integrals

$$\textcircled{1} \int_1^{+\infty} \frac{1}{x^2} dx \quad \textcircled{4} \int_0^1 \frac{1}{x^2} dx$$

$$\textcircled{2} \int_1^{+\infty} \frac{1}{\sqrt{x}} dx \quad \textcircled{5} \int_0^1 \frac{1}{\sqrt{x}} dx$$

$$\textcircled{3} \int_1^{+\infty} \frac{1}{x^2 + \sqrt{x}} dx \quad \textcircled{6} \int_0^1 \frac{1}{x^2 + \sqrt{x}} dx$$

For 3, 6 and 9: draw the graphs of $y = \sqrt{x}$ and $y = x^2$.

Rapid questions: review of improper integrals

$$1 \quad \int_1^{+\infty} \frac{1}{x^2} dx$$

$$4 \quad \int_0^1 \frac{1}{x^2} dx$$

$$7 \quad \int_0^{+\infty} \frac{1}{x^2} dx$$

$$2 \quad \int_1^{+\infty} \frac{1}{\sqrt{x}} dx$$

$$5 \quad \int_0^1 \frac{1}{\sqrt{x}} dx$$

$$8 \quad \int_0^{+\infty} \frac{1}{\sqrt{x}} dx$$

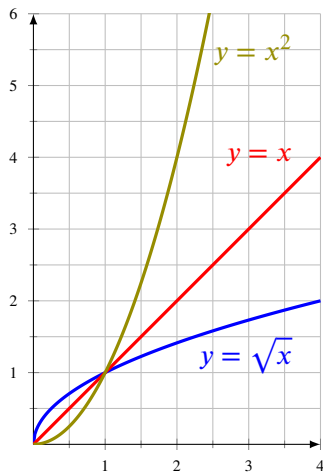
$$3 \quad \int_1^{+\infty} \frac{1}{x^2 + \sqrt{x}} dx$$

$$6 \quad \int_0^1 \frac{1}{x^2 + \sqrt{x}} dx$$

$$9 \quad \int_0^{+\infty} \frac{1}{x^2 + \sqrt{x}} dx$$

For 3, 6 and 9: draw the graphs of $y = \sqrt{x}$ and $y = x^2$.

Some graphs for the previous slide



Trig series: convergent or divergent?

$$① \sum_{n=0}^{\infty} \sin(n\pi)$$

$$② \sum_{n=0}^{\infty} \cos(n\pi)$$

A telescopic series

Goal: Compute $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$.

- 1 Find a formula for the k -th partial sum $S_k = \sum_{n=1}^k \frac{1}{n^2 + 2n}$.

Hint: Write $\frac{1}{n^2 + n} = \frac{A}{n} + \frac{B}{n + 2}$

- 2 Using the definition of series, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$$

Harmonic series

For each $n > 0$ we define

$r_n =$ the smallest power of 2 that is greater than or equal to n

Consider the series $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$

- 1 Compute r_1 through r_8
- 2 Compute the partial sums S_1, S_2, S_4, S_8 for the series S .

- 3 Compute $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$.

- 4 Compute $H = \sum_{n=1}^{\infty} \frac{1}{n}$.

Hint: “Compare” H and S .