MAT137Y1 – LEC0501

Calculus!

Definition of series

March 4^{th}, 2019

For next lecture

For Wednesday (Mar 6), watch the videos:

- Properties of series: 13.5, 13.6, 13.7, 13.8, 13.9

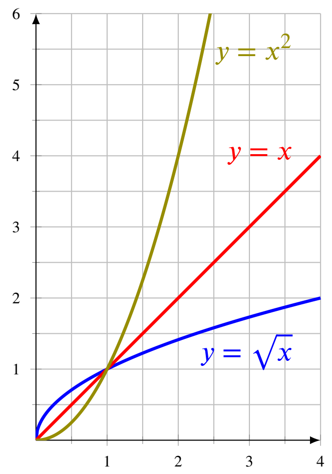
Comment: I wrote some notes about last lecture.
http://www.math.toronto.edu/campesat/ens/1819/lec38-notes.pdf

Rapid questions: review of improper integrals

1. $\int_1^{+\infty} \frac{1}{x^2}\,dx$
2. $\int_1^{+\infty} \frac{1}{\sqrt{x}}\,dx$
3. $\int_1^{+\infty} \frac{1}{x^2+\sqrt{x}}\,dx$
4. $\int_0^{1} \frac{1}{x^2}\,dx$
5. $\int_0^{1} \frac{1}{\sqrt{x}}\,dx$
6. $\int_0^{1} \frac{1}{x^2+\sqrt{x}}\,dx$
7. $\int_0^{+\infty} \frac{1}{x^2}\,dx$
8. $\int_0^{+\infty} \frac{1}{\sqrt{x}}\,dx$
9. $\int_0^{+\infty} \frac{1}{x^2+\sqrt{x}}\,dx$

For 3, 6 and 9: draw the graphs of $y=\sqrt{x}$ *and* $y=x^2$.

Some graphs for the previous slide



Trig series: convergent or divergent?

1. $\sum_{n=0}^{\infty} \sin(n\pi)$
2. $\sum_{n=0}^{\infty} \cos(n\pi)$

A telescopic series

Goal: Compute $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$.

1. Find a formula for the k-th partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n^2+2n}$.

   Hint: Write $\frac{1}{n^2+n} = \frac{A}{n} + \frac{B}{n+2}$

2. Using the definition of series, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$$

Harmonic series

For each $n > 0$ we define

$r_n =$ the smallest power of 2 that is greater than or equal to n

Consider the series $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$

1. Compute r_1 through r_8
2. Compute the partial sums S_1, S_2, S_4, S_8 for the series S.
3. Compute $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$.
4. Compute $H = \sum_{n=1}^{\infty} \frac{1}{n}$.

   Hint: “Compare” H and S.